Session 2.

SCALAR AND VECTOR FIELDS

$$h(x, y) = 6 - 2\sqrt{x^2 + y^2} \qquad r(x, y, z) = \sqrt{x^2 + y^2 + z^2}$$

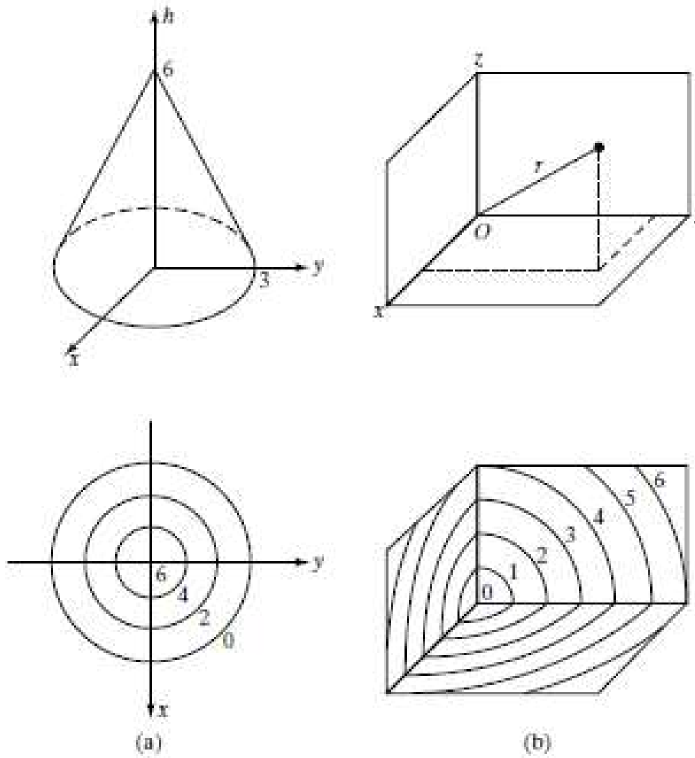


FIGURE 1.17

(a) Conical pyramid lying above the xy-plane and a set of constant-height contours for the conical surface. (b) Rectangular room and a set of constant-distance surfaces depicting the distance field of points in the room from one corner of the room.

$$\boxed{\mathbf{F}(x, y, z, t) = F_x(x, y, z, t)\mathbf{a}_x + F_y(x, y, z, t)\mathbf{a}_y + F_z(x, y, z, t)\mathbf{a}_z} \qquad (1.52)$$

Similar expressions in cylindrical and spherical coordinate systems are as follows:

$$\mathbf{F}(r, \phi, z, t) = F_r(r, \phi, z, t)\mathbf{a}_r + F_\phi(r, \phi, z, t)\mathbf{a}_\phi + F_z(r, \phi, z, t)\mathbf{a}_z \qquad (1.53a)$$

$$\mathbf{F}(r, \theta, \phi, t) = F_r(r, \theta, \phi, t)\mathbf{a}_r + F_\theta(r, \theta, \phi, t)\mathbf{a}_\theta + F_\phi(r, \theta, \phi, t)\mathbf{a}_\phi \qquad (1.53b)$$

Finding equations for direction lines of a vector field

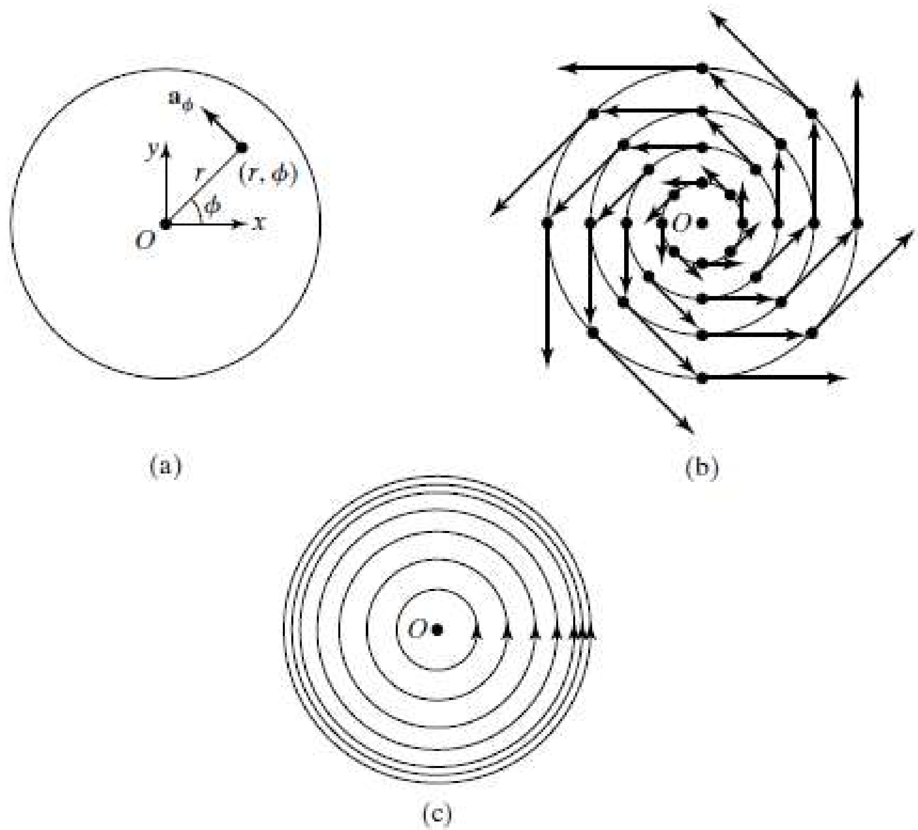
$$\frac{dx}{F_x} = \frac{dy}{F_y} = \frac{dz}{F_z}$$

$$\frac{dr}{F_r} = \frac{r\,d\phi}{F_\phi} = \frac{dz}{F_z}$$
$$\frac{dr}{F_r} = \frac{r\,d\theta}{F_\theta} = \frac{r\sin\theta\,d\phi}{F_\phi}$$

Example 1.5 Linear velocity vector field of points on a rotating disk

Consider a circular disk of radius a rotating with constant angular velocity ω about an axis normal to the disk and passing through its center. We wish to describe the linear velocity vector field associated with points on the rotating disk.

We choose the center of the disk to be the origin and set up a two-dimensional coordinate system, as shown in Fig. 1.18(a). Note that we have a choice of the coordinates



D1.10. The time-varying temperature field in a certain region of space is given by

$$T(x, y, z, t) = T_0\{[x(1 + \sin \pi t)]^2 + [2y(1 - \cos \pi t)]^2 + 4z^2\}$$

where T_0 is a constant. Find the shapes of the constant-temperature surfaces for each of the following values of t: **(a)** $t = 0$; **(b)** $t = 0.5$ s; and **(c)** $t = 1$ s.

Ans. **(a)** elliptic cylinders; **(b)** spheres; **(c)** ellipsoids.

D1.11. For the vector field $\mathbf{F} = (3x - y)\mathbf{a}_x + (x + z)\mathbf{a}_y + (2y - z)\mathbf{a}_z$, find the following: **(a)** the magnitude of $\mathbf{F}$ and the unit vector along $\mathbf{F}$ at the point $(1, 1, 0)$; **(b)** the point at which the magnitude of $\mathbf{F}$ is 3 and the direction of $\mathbf{F}$ is along the unit vector $\frac{1}{3}(2\mathbf{a}_x + 2\mathbf{a}_y + \mathbf{a}_z)$; and **(c)** the point at which the magnitude of $\mathbf{F}$ is 3 and the direction of $\mathbf{F}$ is along the unit vector $\mathbf{a}_z$.

Ans. **(a)** $3, \frac{1}{3}(2\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)$; **(b)** $(1, 1, 1)$; **(c)** $(0.6, 1.8, -0.6)$.

D1.12. A vector field is given in cylindrical coordinates by

$$\mathbf{F} = \frac{1}{r^2}(\cos \phi\, \mathbf{a}_r + \sin \phi\, \mathbf{a}_\phi)$$

Express the vector $\mathbf{F}$ in Cartesian coordinates at each of the following points specified in Cartesian coordinates: **(a)** $(1, 0, 0)$; **(b)** $(1, -1, -3)$; and **(c)** $(1, \sqrt{3}, -4)$.

Ans. **(a)** $\mathbf{a}_x$; **(b)** $-\frac{1}{2}\mathbf{a}_y$; **(c)** $\frac{1}{8}(-\mathbf{a}_x + \sqrt{3}\mathbf{a}_y)$.

THE ELECTRIC FIELD

For free space, the constant of proportionality is $1/4\pi\varepsilon_0$, where ε_0 is known as the permittivity of free space, having a value 8.854×10^{-12}, or approximately equal to $10^{-9}/36\pi$. (For convenience, we shall use a value of $10^{-9}/36\pi$ for ε_0 throughout this book.) Thus, if we consider two point charges Q_1 C and Q_2 C separated R m in free space, as shown in Fig. 1.19, then the forces $\mathbf{F}_1$ and $\mathbf{F}_2$ experienced by Q_1 and Q_2, respectively, are given by

$$\boxed{\mathbf{F}_1 = \frac{Q_1Q_2}{4\pi\varepsilon_0 R^2}\mathbf{a}_{21}} \tag{1.57a}$$

and

$$\boxed{\mathbf{F}_2 = \frac{Q_2Q_1}{4\pi\varepsilon_0 R^2}\mathbf{a}_{12}} \tag{1.57b}$$

$$\mathbf{E} = \frac{\mathbf{F}}{q} \tag{1.58}$$

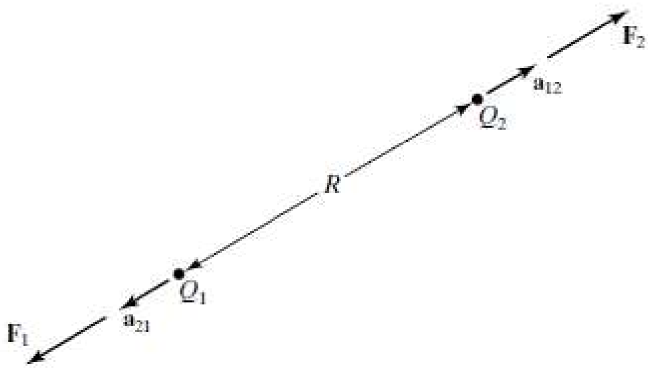


FIGURE 1.19

Forces experienced by two point charges Q_1 and Q_2.

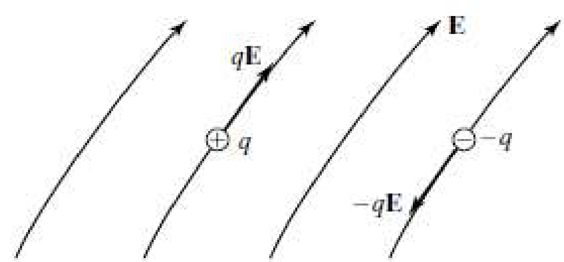


FIGURE 1.20

Forces experienced by positive and negative charges in an electric field.

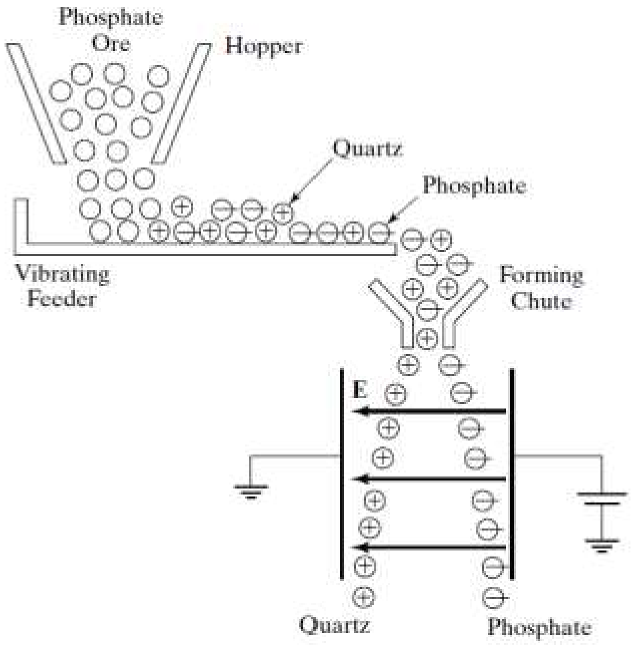


FIGURE 1.21

Example for illustrating electrostatic separation of minerals.

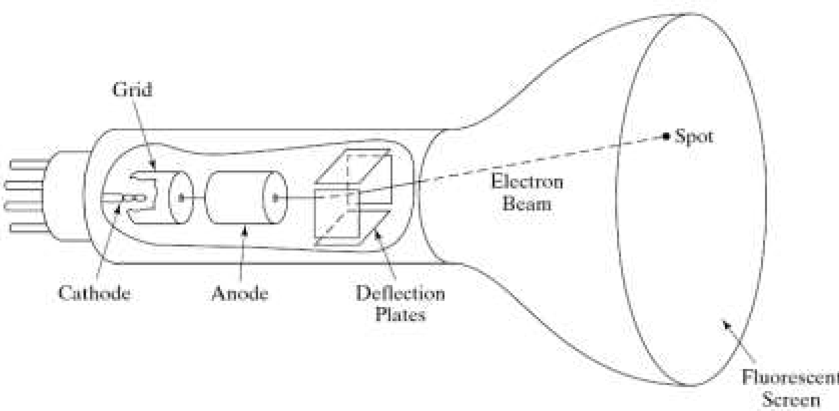


FIGURE 1.22

Schematic diagram of a cathode ray tube.