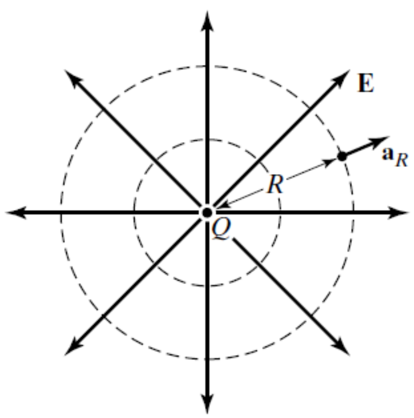
Session 3

Electric field due to a point charge

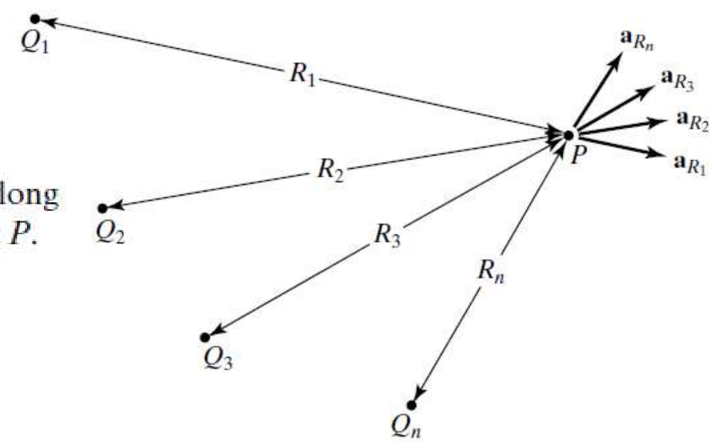
$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R$$

Q located at a point $P'(x', y', z')$

$$\mathbf{E} = \frac{Q\mathbf{R}}{4\pi\varepsilon_0 R^3} = \frac{Q}{4\pi\varepsilon_0}\frac{(x - x')\mathbf{a}_x + (y - y')\mathbf{a}_y + (z - z')\mathbf{a}_z}{[(x - x')^2 + (y - y')^2 + (z - z')^2]^{3/2}}$$



Direction lines and cross sections of constant-magnitude surfaces of electric field due to a point charge.



Collection of point charges and unit vectors along the directions of their electric fields at a point P.

$$\mathbf{E} = \frac{Q_1}{4\pi\varepsilon_0 R_1^2}\mathbf{a}_{R_1} + \frac{Q_2}{4\pi\varepsilon_0 R_2^2}\mathbf{a}_{R_2} + \cdots + \frac{Q_n}{4\pi\varepsilon_0 R_n^2}\mathbf{a}_{R_n}$$

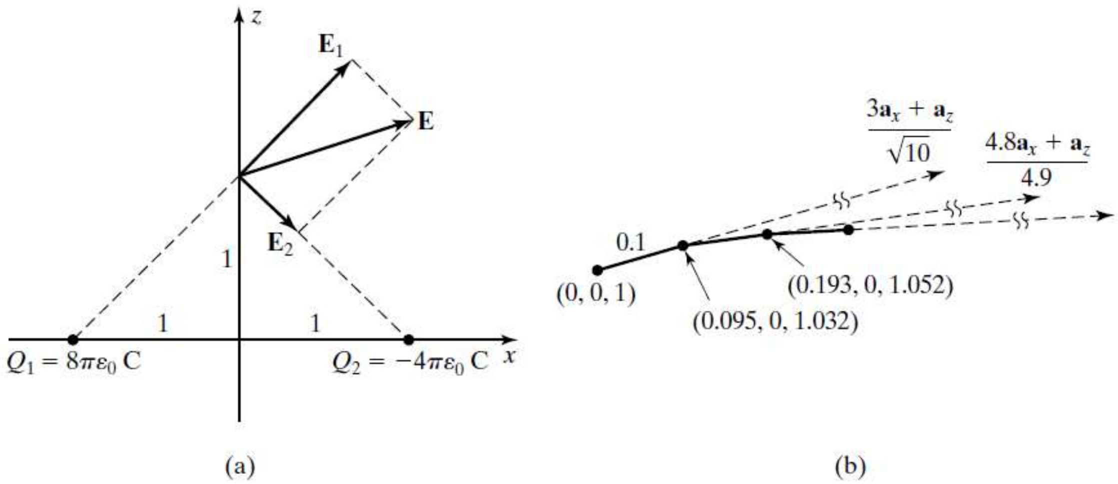


FIGURE 1.25

(a) Computation of the resultant electric field due to two point charges. (b) Generation of the direction line of the electric field of (a).

TABLE 1.1 Values of Parameters Pertinent to the Steps in the Computer Generation of the Direction Line of **E** in Fig. 1.25 for (a) the Segment from (0, 0, 1) Toward the Charge Q_2 and (b) the Segment from (0, 0, 1) Back Toward the Charge Q_1.

X = 0.000	Z = 1.000	E = 1.118	UX = 0.949	UZ = 0.316
X = 0.095	Z = 1.032	E = 1.015	UX = 0.979	UZ = 0.204
X = 0.193	Z = 1.052	E = 0.942	UX = 0.997	UZ = 0.076
X = 0.292	Z = 1.060	E = 0.898	UX = 0.998	UZ = −0.065
X = 0.392	Z = 1.053	E = 0.882	UX = 0.977	UZ = −0.215
X = 0.490	Z = 1.032	E = 0.898	UX = 0.930	UZ = −0.368
X = 0.583	Z = 0.995	E = 0.951	UX = 0.858	UZ = −0.513
X = 0.669	Z = 0.944	E = 1.051	UX = 0.766	UZ = −0.643
X = 0.745	Z = 0.879	E = 1.212	UX = 0.660	UZ = −0.751
X = 0.811	Z = 0.804	E = 1.459	UX = 0.548	UZ = −0.836
X = 0.866	Z = 0.721	E = 1.837	UX = 0.439	UZ = −0.899
X = 0.910	Z = 0.631	E = 2.426	UX = 0.337	UZ = −0.942
X = 0.944	Z = 0.536	E = 3.391	UX = 0.246	UZ = −0.969
X = 0.968	Z = 0.440	E = 5.100	UX = 0.167	UZ = −0.986
X = 0.985	Z = 0.341	E = 8.537	UX = 0.101	UZ = −0.995
X = 0.995	Z = 0.241	E = 17.101	UX = 0.049	UZ = −0.999
X = 1.000	Z = 0.142	E = 49.846	UX = 0.010	UZ = −1.000
X = 1.001	Z = 0.042	E = 577.540	UX = −0.023	UZ = −1.000
Number of steps = 17				

(a)

X = 0.000	Z = 1.000	E = 1.118	UX = 0.949	UZ = 0.316
X = −.095	Z = 0.968	E = 1.243	UX = 0.908	UZ = 0.420
X = −.186	Z = 0.926	E = 1.411	UX = 0.862	UZ = 0.507
X = −.272	Z = 0.876	E = 1.634	UX = 0.815	UZ = 0.580
X = −.353	Z = 0.818	E = 1.931	UX = 0.768	UZ = 0.640
X = −.430	Z = 0.754	E = 2.333	UX = 0.724	UZ = 0.689
X = −.503	Z = 0.685	E = 2.888	UX = 0.684	UZ = 0.730
X = −.571	Z = 0.612	E = 3.681	UX = 0.648	UZ = 0.762
X = −.636	Z = 0.536	E = 4.871	UX = 0.616	UZ = 0.788
X = −.697	Z = 0.457	E = 6.769	UX = 0.590	UZ = 0.808
X = −.756	Z = 0.376	E = 10.074	UX = 0.568	UZ = 0.823
X = −.813	Z = 0.294	E = 16.616	UX = 0.551	UZ = 0.835
X = −.868	Z = 0.210	E = 32.588	UX = 0.538	UZ = 0.843
X = −.922	Z = 0.126	E = 91.176	UX = 0.529	UZ = 0.849
X = −.975	Z = 0.041	E = 860.610	UX = 0.522	UZ = 0.853
Number of steps = 14				

(b)

Table 1.1, parts (a) and (b), corresponding to the segments of the direction line from (0, 0, 1) toward Q_2 and Q_1, respectively. It can be seen that the test charge takes 17 steps toward Q_2 but only 14 steps back toward Q_1.

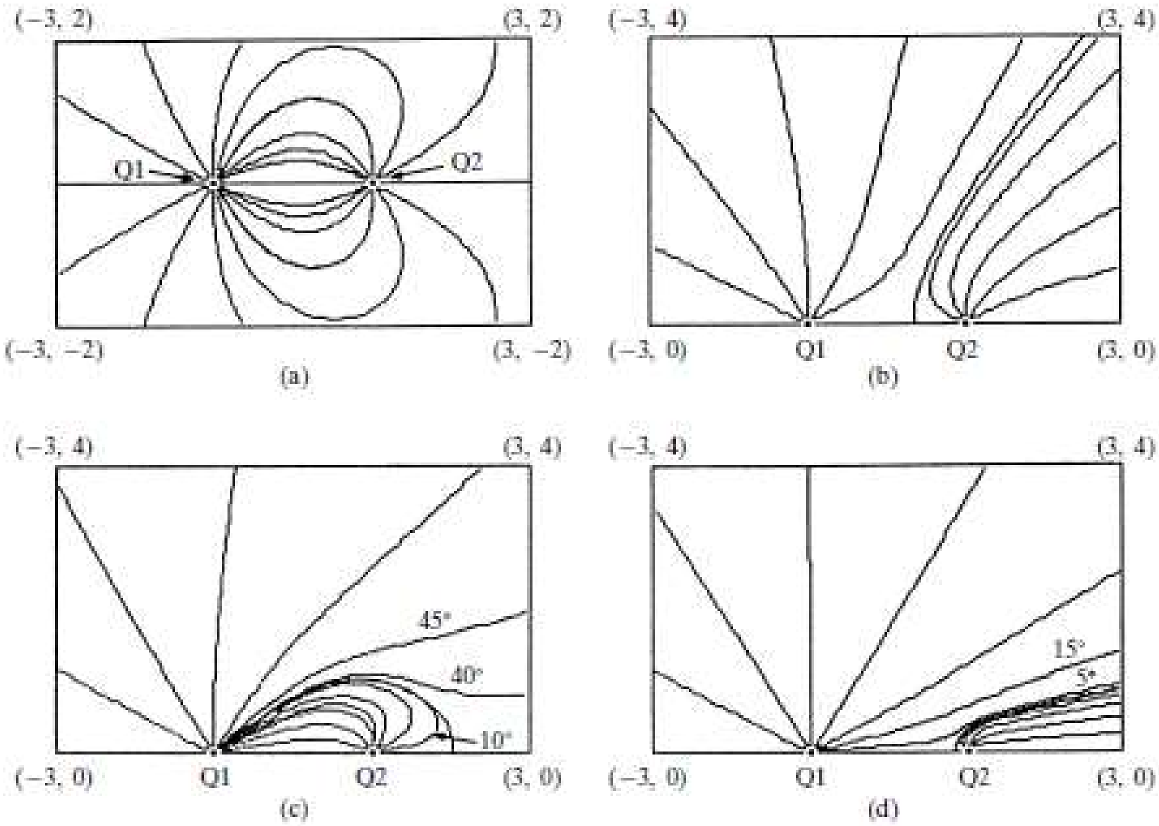


FIGURE 1.26

Computer-generated maps of direction lines of electric field for pairs of point charges Q_1 and Q_2 at $(-1, 0)$ and $(1, 0)$, respectively, in the xz-plane. (a) $Q_1 = 2Q, Q_2 = -Q$; (b) $Q_1 = 4Q, Q_2 = Q$; (c) $Q_1 = 9Q, Q_2 = -Q$; and (d) $Q_1 = 81Q, Q_2 = Q$.

Example 1.7 Circular ring charge with uniform density

$$
\begin{aligned}
[E_z]_{(0,0,z)} &= \int_{\phi=0}^{\pi} \frac{2(Q/2\pi a)a\,d\phi}{4\pi\varepsilon_0(a^2 + z^2)} \frac{z}{(a^2 + z^2)^{1/2}} \\
&= \frac{Qz}{4\pi^2\varepsilon_0(a^2 + z^2)^{3/2}} \int_{\phi=0}^{\pi} d\phi \\
&= \frac{Qz}{4\pi\varepsilon_0(a^2 + z^2)^{3/2}}
\end{aligned}
$$

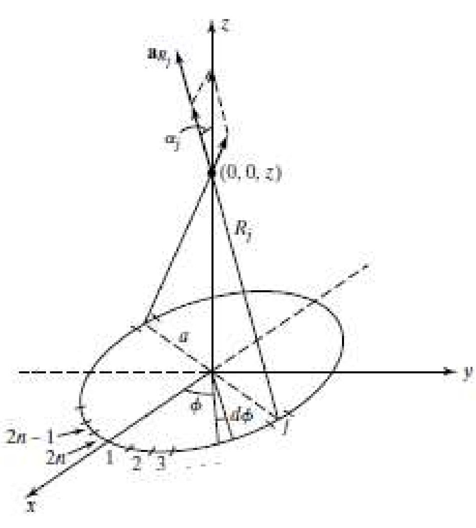


FIGURE 1.27

Determination of electric field due to a circular ring of charge of uniform density.

Example 1.8 Electric field of an infinitely long line charge of uniform density

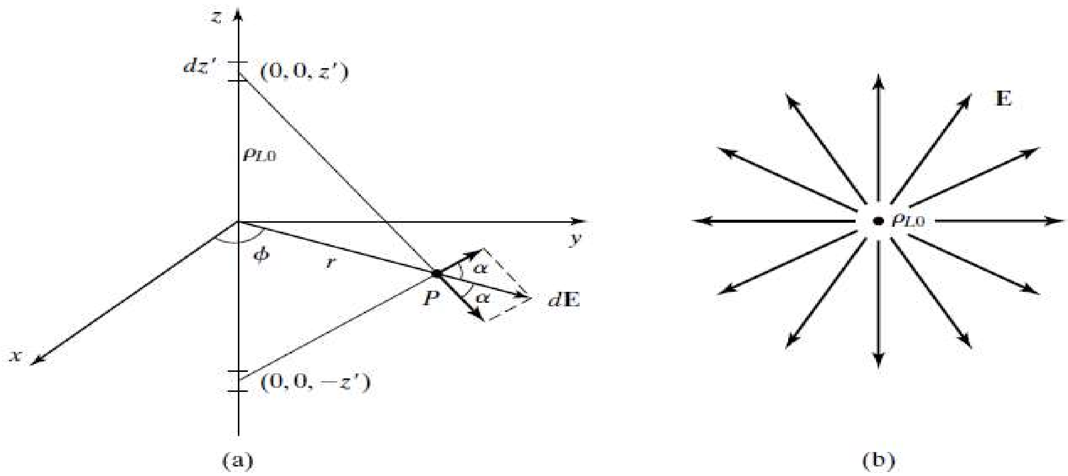


FIGURE 1.28

(a) Determination of electric field due to an infinitely long line charge of uniform charge density ρ_{L0} C/m. (b) Electric field due to the infinitely long line charge of (a).

$$[d\mathbf{E}]_{(r,\phi,0)} = 2\frac{\rho_{L0}\,dz'}{4\pi\varepsilon_0[r^2 + (z')^2]}\cos\alpha\,\mathbf{a}_r$$
$$= \frac{\rho_{L0}r\,dz'}{2\pi\varepsilon_0[r^2 + (z')^2]^{3/2}}\mathbf{a}_r$$

The electric field intensity at P due to the entire line charge is then given by

$$[\mathbf{E}]_{(r,\phi,0)} = \int_{z'=0}^{\infty}[d\mathbf{E}]_{(r,\phi,0)}$$
$$= \int_{z'=0}^{\infty}\frac{\rho_{L0}r\,dz'}{2\pi\varepsilon_0[r^2 + (z')^2]^{3/2}}\mathbf{a}_r$$
$$= \frac{\rho_{L0}}{2\pi\varepsilon_0 r}\int_{\alpha=0}^{\pi/2}\cos\alpha\,d\alpha$$
$$= \frac{\rho_{L0}}{2\pi\varepsilon_0 r}\mathbf{a}_r$$

Example 1.9 Electric field of an infinite plane sheet of charge of uniform density

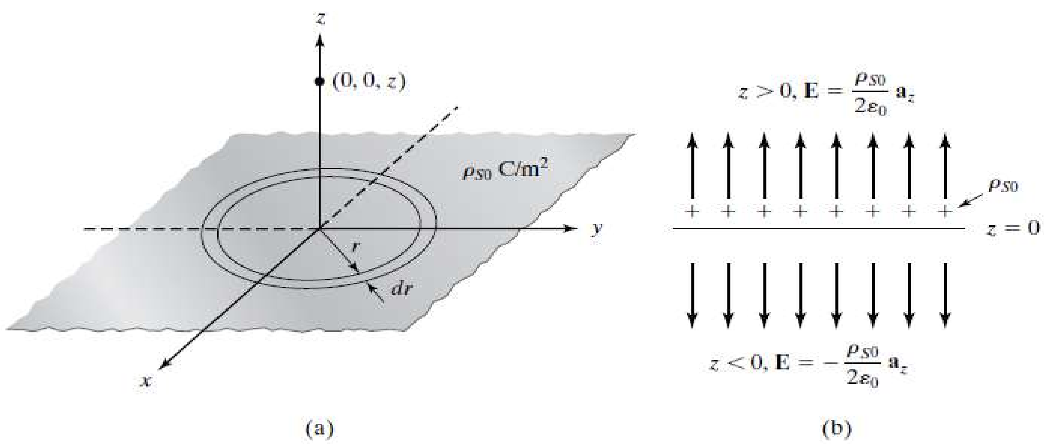


FIGURE 1.29

(a) Determination of electric field due to an infinite plane sheet of uniform surface charge density ρ_{S0} C/m^2. (b) Electric field due to the infinite plane sheet of charge of (a).

$$[d\mathbf{E}]_{(0,0,z)} = \frac{(\rho_{S0} 2\pi r\, dr)z}{4\pi\varepsilon_0(r^2 + z^2)^{3/2}}\mathbf{a}_z$$

The electric field intensity due to the entire sheet of charge is then given by

$$\begin{aligned}[\mathbf{E}]_{(0,0,z)} &= \int_{r=0}^{\infty} [d\mathbf{E}]_{(0,0,z)} \\ &= \int_{r=0}^{\infty} \frac{\rho_{S0} rz\, dr}{2\varepsilon_0(r^2 + z^2)^{3/2}}\mathbf{a}_z \\ &= \frac{\rho_{S0} z}{2\varepsilon_0}\left[-\frac{1}{\sqrt{r^2 + z^2}}\right]_{r=0}^{\infty}\mathbf{a}_z \\ &= \frac{\rho_{S0} z}{2\varepsilon_0 |z|}\mathbf{a}_z\end{aligned}$$

$$\mathbf{a}_n = \pm\mathbf{a}_z \quad \text{for} \quad z \gtrless 0$$

$$\boxed{\mathbf{E} = \frac{\rho_{S0}}{2\varepsilon_0}\mathbf{a}_n}$$

D1.13. Point charges, each of value $\sqrt{4\pi\varepsilon_0}$ C, are located at the vertices of an n-sided regular polygon circumscribed by a circle of radius a. Find the electric force on each charge for **(a)** $n = 3$; **(b)** $n = 4$; and **(c)** $n = 6$.

Ans. **(a)** $0.577/a^2$ N; **(b)** $0.957/a^2$ N; **(c)** $1.827/a^2$ N; all directed away from the center of the polygon.

D1.14. In Fig. 1.25, let the point charges be $Q_1 = 8\pi\varepsilon_0$ C at $(-1, 0, 0)$ and $Q_2 = 4\pi\varepsilon_0$ C at $(1, 0, 0)$. Find the following: **(a)** **E** at $(0, 0, 1)$; **(b)** the coordinates of the point at the end of the second step; and **(c)** the unit vector along **E** at the point computed in **(b)**.

Ans. **(a)** $(0.353\mathbf{a}_x + 1.061\mathbf{a}_z)$; **(b)** $(0.060, 0, 1.191)$; **(c)** $(0.264\mathbf{a}_x + 0.965\mathbf{a}_z)$.

D1.15. In Fig. 1.27, let there be a second ring of charge $-Q$, uniformly distributed along a circle of radius a, having its center at $(0, 0, 2a)$ and lying parallel to the xy-plane. Find **E** due to the two rings of charge together at each of the following points: **(a)** $(0, 0, 0)$; **(b)** $(0, 0, a)$; and **(c)** $(0, 0, 3a)$.

Ans. **(a)** $(0.0142Q/\varepsilon_0 a^2)\mathbf{a}_z$; **(b)** $(0.0563Q/\varepsilon_0 a^2)\mathbf{a}_z$; **(c)** $(-0.0206Q/\varepsilon_0 a^2)\mathbf{a}_z$.

D1.16. Infinite plane sheets of charge lie in the $z = 0$, $z = 2$, and $z = 4$ planes with uniform surface charge densities ρ_{S1}, ρ_{S2}, and ρ_{S3}, respectively. Given that the resulting electric field intensities at the points $(3, 5, 1)$, $(1, -2, 3)$, and $(3, 4, 5)$ are $\mathbf{0}$, $6\mathbf{a}_z$, and $4\mathbf{a}_z$ V/m, respectively, find the following: **(a)** ρ_{S1}; **(b)** ρ_{S2}; **(c)** ρ_{S3}; and **(d)** **E** at $(-2, 1, -6)$.

Ans. **(a)** $4\varepsilon_0$ C/m^2; **(b)** $6\varepsilon_0$ C/m^2; **(c)** $-2\varepsilon_0$ C/m^2; **(d)** $-4\mathbf{a}_z$ V/m.