Session 4 :

THE MAGNETIC FIELD

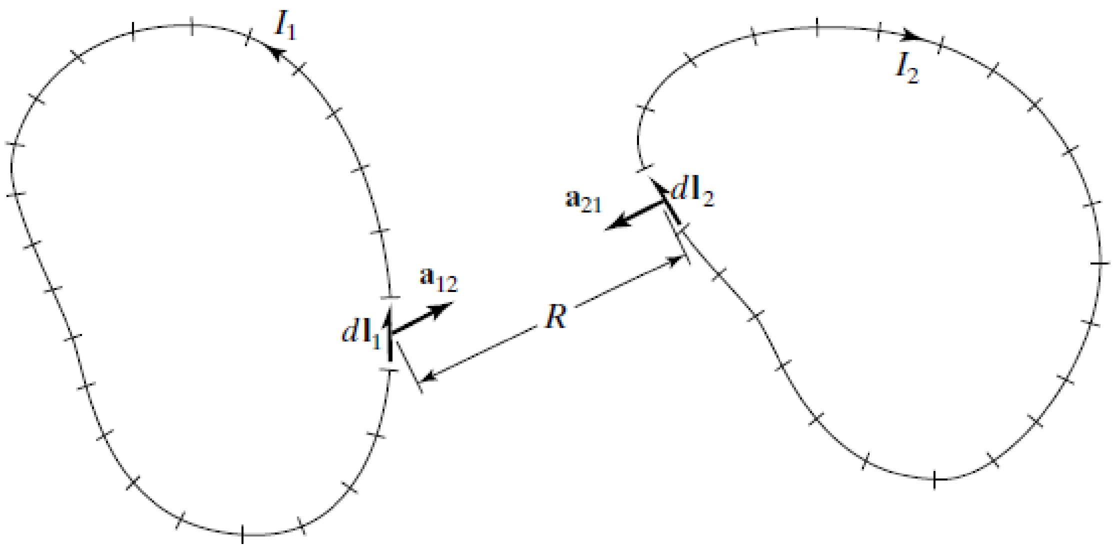


FIGURE 1.30

Two loops of wire carrying currents I_1 and I_2.

$$d\mathbf{F}_1 = I_1\, d\mathbf{l}_1 \times \left(\frac{kI_2\, d\mathbf{l}_2 \times \mathbf{a}_{21}}{R^2} \right)$$

$$d\mathbf{F}_2 = I_2\, d\mathbf{l}_2 \times \left(\frac{kI_1\, d\mathbf{l}_1 \times \mathbf{a}_{12}}{R^2} \right)$$

Magnetic flux density

$$\mathbf{B}_1 = \frac{\mu_0}{4\pi} \frac{I_1\, d\mathbf{l}_1 \times \mathbf{a}_{12}}{R^2} \qquad d\mathbf{F}_2 = I_2\, d\mathbf{l}_2 \times \mathbf{B}_1$$

$$\mathbf{B}_2 = \frac{\mu_0}{4\pi} \frac{I_2\, d\mathbf{l}_2 \times \mathbf{a}_{21}}{R^2} \qquad d\mathbf{F}_1 = I_1\, d\mathbf{l}_1 \times \mathbf{B}_2$$

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{I\, d\mathbf{l} \times \mathbf{a}_R}{R^2}$$

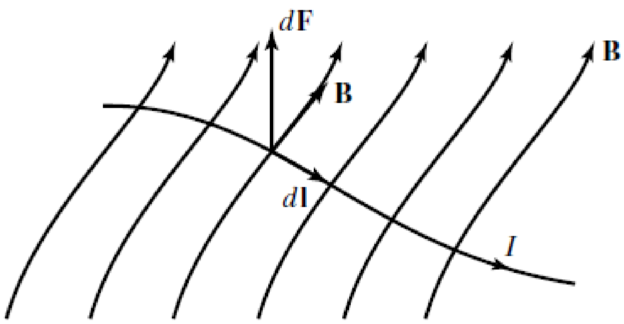
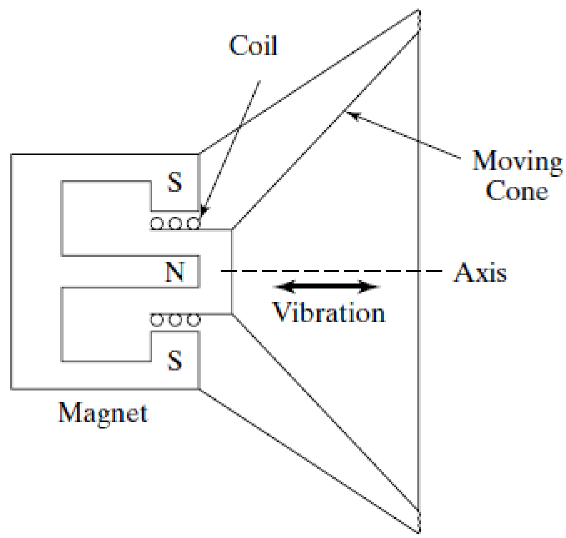


FIGURE 1.31

Force experienced by a current element in a magnetic field.



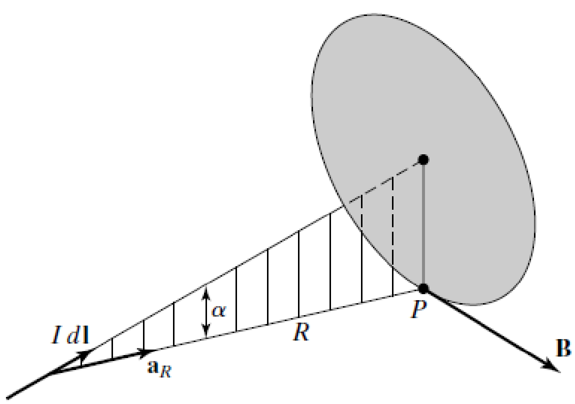


FIGURE 1.33

Magnetic flux density due to an infinitesimal current element.

Magnetic flux density due to a current element

$$I\,d\mathbf{l} = (2)(10^{-3})\mathbf{a}_x = 0.002\mathbf{a}_x$$

$$\mathbf{R} = (0 - 1)\mathbf{a}_x + (2 - 0)\mathbf{a}_y + (2 - 0)\mathbf{a}_z = -\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z$$

$$\begin{aligned}[\mathbf{B}]_{(0,2,2)} &= \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{l} \times \mathbf{a}_R}{R^2} \\ &= \frac{\mu_0}{4\pi}\frac{I\,d\mathbf{l} \times \mathbf{R}}{R^3} \\ &= \frac{\mu_0}{4\pi}\frac{0.002\mathbf{a}_x \times (-\mathbf{a}_x + 2\mathbf{a}_y + 2\mathbf{a}_z)}{27} \\ &= \frac{0.001\mu_0}{27\pi}(-\mathbf{a}_y + \mathbf{a}_z)\ \text{Wb/m}^2\end{aligned}$$

Example 1.11 Magnetic field of an infinitely long straight wire of current

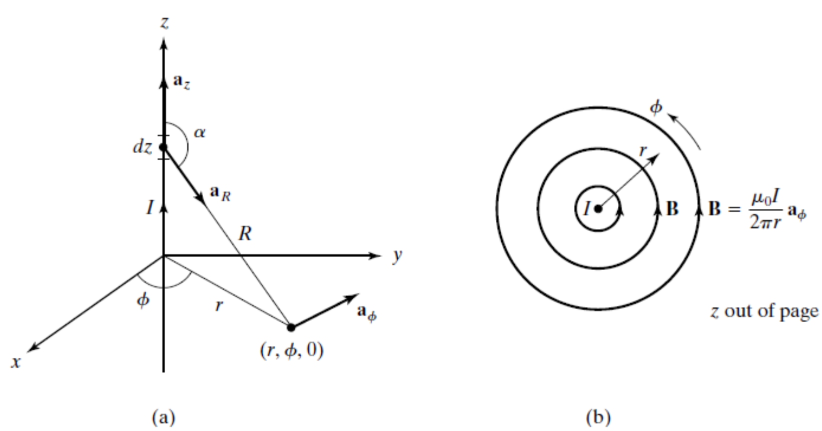


FIGURE 1.34

(a) Determination of magnetic field due to an infinitely long, straight wire of current I A.
(b) Magnetic field due to the wire of (a).

$$[d\mathbf{B}]_{(r,\phi,0)} = \frac{\mu_0}{4\pi}\frac{I\,dz\,\mathbf{a}_z \times \mathbf{a}_R}{R^2}$$
$$= \frac{\mu_0 I\,dz}{4\pi}\frac{\sin\alpha}{R^2}\mathbf{a}_\phi$$
$$= \frac{\mu_0 I\,dz}{4\pi}\frac{r}{R^3}\mathbf{a}_\phi$$
$$= \frac{\mu_0 I r\,dz}{4\pi(z^2 + r^2)^{3/2}}\mathbf{a}_\phi$$

$$[\mathbf{B}]_{(r,\phi,0)} = \int_{z=-\infty}^{\infty} d\mathbf{B}$$
$$= \int_{z=-\infty}^{\infty} \frac{\mu_0 I r}{4\pi(z^2 + r^2)^{3/2}}dz\,\mathbf{a}_\phi$$
$$= \frac{\mu_0 I r}{4\pi}\left[\frac{z}{r^2\sqrt{z^2 + r^2}}\right]_{z=-\infty}^{\infty}\mathbf{a}_\phi$$
$$= \frac{\mu_0 I}{2\pi r}\mathbf{a}_\phi$$

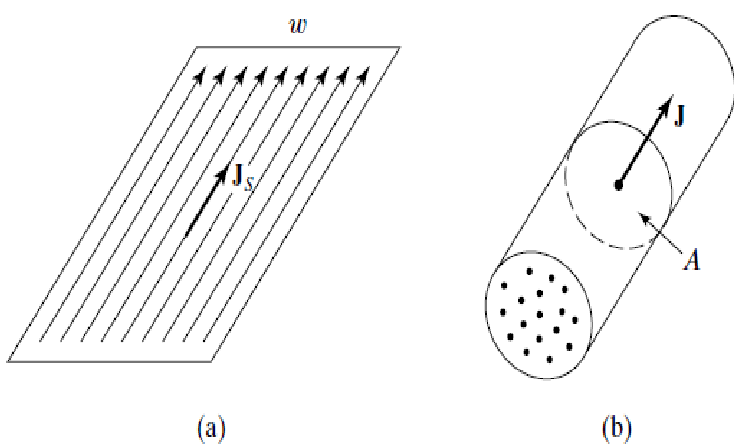


FIGURE 1.35

Determination of currents due to (a) surface current and (b) volume current distributions of uniform densities.

Magnetic field of an infinite plane sheet of current

$$d\mathbf{B} = d\mathbf{B}_1 + d\mathbf{B}_2 = -2\,dB_1\cos\alpha\,\mathbf{a}_x$$
$$= -2\frac{\mu_0 J_{S0}\,dx}{2\pi\sqrt{x^2 + y^2}}\frac{y}{\sqrt{x^2 + y^2}}\mathbf{a}_x = -\frac{\mu_0 J_{S0} y\,dx}{\pi(x^2 + y^2)}\mathbf{a}_x$$

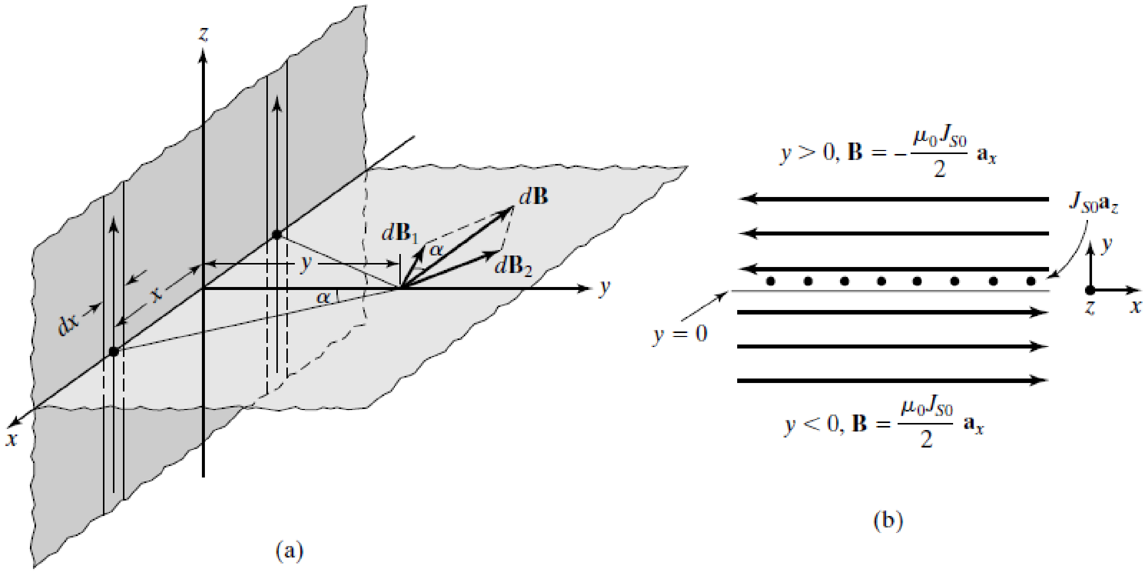


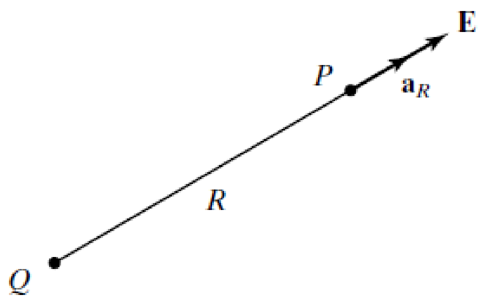
FIGURE 1.36

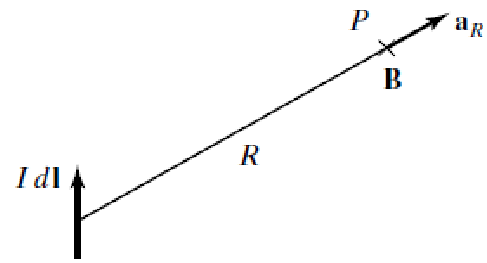
(a) Determination of magnetic field due to an infinite plane sheet of current density $J_{S0}\mathbf{a}_z$ A/m. (b) Magnetic field due to the current sheet of (a).

$$
\begin{aligned}
[\mathbf{B}]_{(0,\,y,\,0)} &= \int_{x=0}^{\infty} d\mathbf{B} \\
&= -\int_{x=0}^{\infty} \frac{\mu_0 J_{S0} y}{\pi(x^2 + y^2)}\,dx\,\mathbf{a}_x \\
&= -\frac{\mu_0 J_{S0} y}{\pi}\left[\frac{1}{y}\tan^{-1}\frac{x}{y}\right]_{x=0}^{\infty}\mathbf{a}_x \\
&= -\frac{\mu_0 J_{S0}}{2}\mathbf{a}_x \qquad \text{for} \quad y > 0
\end{aligned}
$$

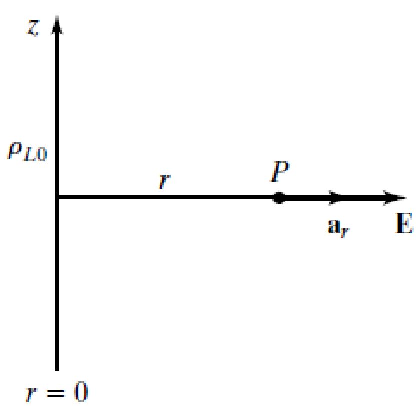
$$
\boxed{\mathbf{B} = \frac{\mu_0}{2}\mathbf{J}_S \times \mathbf{a}_n}
$$

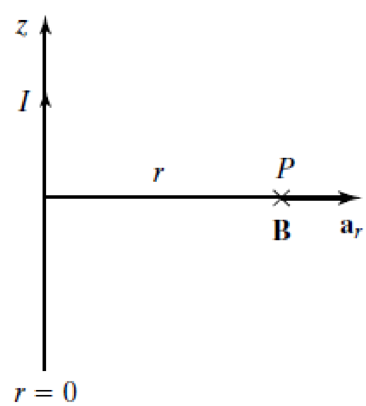
$$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\mathbf{a}_R \qquad \leftrightarrow \qquad \mathbf{B} = \frac{\mu_0 I}{4\pi R^2} d\mathbf{l} \times \mathbf{a}_R$$



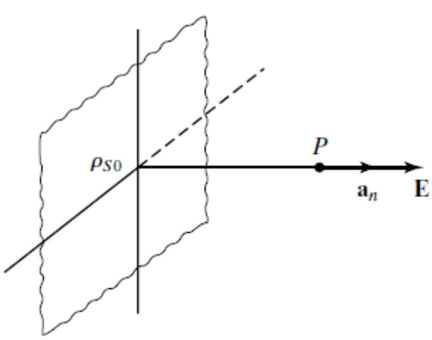


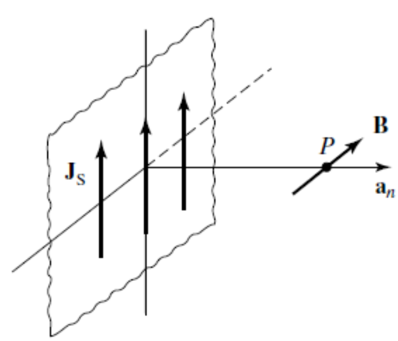
$$\mathbf{E} = \frac{\rho_{L0}}{2\varepsilon_0}\mathbf{a}_n \qquad \leftrightarrow \qquad \mathbf{B} = \frac{\mu_0}{2}\mathbf{J}_S \times \mathbf{a}_n$$

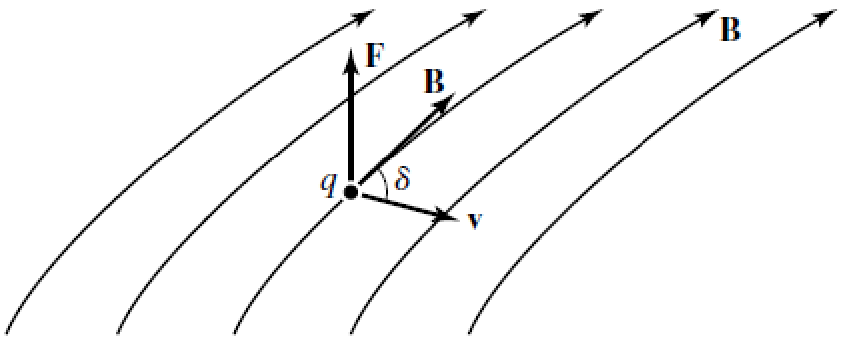




$$\mathbf{E} = \frac{\rho_{L0}}{2\varepsilon_0}\mathbf{a}_n \qquad \leftrightarrow \qquad \mathbf{B} = \frac{\mu_0}{2}\mathbf{J}_S \times \mathbf{a}_n$$







$$\mathbf{B} = \frac{\mathbf{F}_m \times \mathbf{a}_m}{qv}$$

D1.17. For $I_1\, d\mathbf{l}_1 = I_1\, dy\, \mathbf{a}_y$ located at $(1, 0, 0)$ and $I_2\, d\mathbf{l}_2 = I_2\, dx\, \mathbf{a}_x$, located at $(0, 1, 0)$, find: **(a)** $d\mathbf{F}_1$ and **(b)** $d\mathbf{F}_2$.

Ans. **(a)** $-(\mu_0 I_1 I_2/8\sqrt{2}\pi)\, dx\, dy\, \mathbf{a}_x$; **(b)** $-(\mu_0 I_1 I_2/8\sqrt{2}\pi)\, dx\, dy\, \mathbf{a}_y$.

D1.18. A current I flows in a wire along the curve $x = 2y = z^2 + 2$ and in the direction of increasing z. If the wire is situated in a magnetic field $\mathbf{B} = (y\mathbf{a}_x - x\mathbf{a}_y)/(x^2 + y^2)$, find the magnetic force acting on an infinitesimal length of the wire having the projection dz on the z-axis at each of the following points: **(a)** $(2, 1, 0)$; **(b)** $(3, 1.5, 1)$; and **(c)** $(6, 3, 2)$.

Ans. **(a)** $I\, dz\, (2\mathbf{a}_x + \mathbf{a}_y)/5$; **(b)** $I\, dz\, (2\mathbf{a}_x + \mathbf{a}_y - 5\mathbf{a}_z)/7.5$; **(c)** $I\, dz\, (2\mathbf{a}_x + \mathbf{a}_y - 10\mathbf{a}_z)/15$.

D1.19. Given $\mathbf{B} = (B_0/3)(2\mathbf{a}_x + 2\mathbf{a}_y - \mathbf{a}_z)$, find the magnitude of the magnetic force acting on a test charge q moving with velocity v_0 at the point $(2, 2, -1)$ for each of the following paths of the test charge: **(a)** $x = y = -2z$; **(b)** $4x = 4y = z + 9$; and **(c)** $x = y = 2z^2$.

Ans. **(a)** 0; **(b)** qv_0B_0; **(c)** $0.1641qv_0B_0$.

D1.20. Infinite plane sheets of current lie in the $x = 0$, $y = 0$, and $z = 0$ planes with uniform surface current densities $J_{S0}\, \mathbf{a}_z$, $2J_{S0}\, \mathbf{a}_x$, and $-J_{S0}\, \mathbf{a}_x$ A/m, respectively. Find the resulting magnetic flux densities at the following points: **(a)** $(1, 2, 2)$; **(b)** $(2, -2, -1)$; and **(c)** $(-2, 1, -2)$.

Ans. **(a)** $\mu_0 J_{S0}(\mathbf{a}_y + \mathbf{a}_z)$; **(b)** $-\mu_0 J_{S0}\, \mathbf{a}_z$; **(c)** $\mu_0 J_{S0}(-\mathbf{a}_y + \mathbf{a}_z)$.

LORENTZ FORCE EQUATION

$$\mathbf{F} = \mathbf{F}_E + \mathbf{F}_M = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

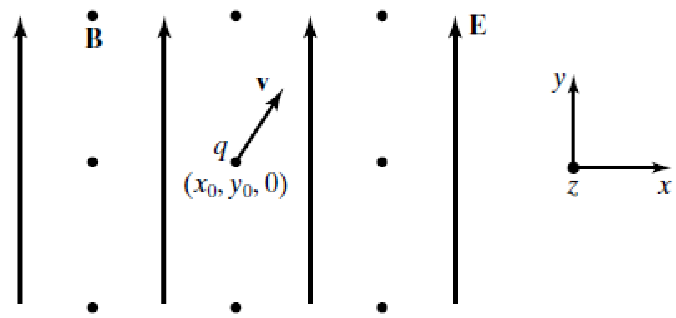


FIGURE 1.40

Test charge q in a region of crossed electric and magnetic fields.

$$\begin{aligned}\mathbf{F} &= q(\mathbf{E} + \mathbf{v} \times \mathbf{B})\\ &= qE_0\mathbf{a}_y + q(v_x\mathbf{a}_x + v_y\mathbf{a}_y + v_z\mathbf{a}_z) \times B_0\mathbf{a}_z\\ &= qB_0v_y\mathbf{a}_x + (qE_0 - qB_0v_x)\mathbf{a}_y\end{aligned}$$

$$\frac{dv_x}{dt} = \frac{qB_0}{m}v_y$$

$$\frac{dv_y}{dt} = \frac{qE_0}{m} - \frac{qB_0}{m}v_x$$

$$\frac{dv_z}{dt} = 0$$

$$\frac{d^2v_x}{dt^2} + \left(\frac{qB_0}{m}\right)^2 v_x = \left(\frac{q}{m}\right)^2 B_0E_0$$

$$v_x = \frac{E_0}{B_0} + C_1 \cos \omega_c t + C_2 \sin \omega_c t \qquad v_y = -C_1 \sin \omega_c t + C_2 \cos \omega_c t$$

$$v_x = \frac{E_0}{B_0} + \left(v_{x0} - \frac{E_0}{B_0}\right)\cos\omega_c t + v_{y0}\sin\omega_c t$$

$$v_y = -\left(v_{x0} - \frac{E_0}{B_0}\right)\sin\omega_c t + v_{y0}\cos\omega_c t$$

$$x = x_0 + \frac{E_0}{B_0}t + \frac{1}{\omega_c}\left(v_{x0} - \frac{E_0}{B_0}\right)\sin\omega_c t + \frac{v_{y0}}{\omega_c}(1 - \cos\omega_c t)$$

$$y = y_0 - \frac{1}{\omega_c}\left(v_{x0} - \frac{E_0}{B_0}\right)(1 - \cos\omega_c t) + \frac{v_{y0}}{\omega_c}\sin\omega_c t$$

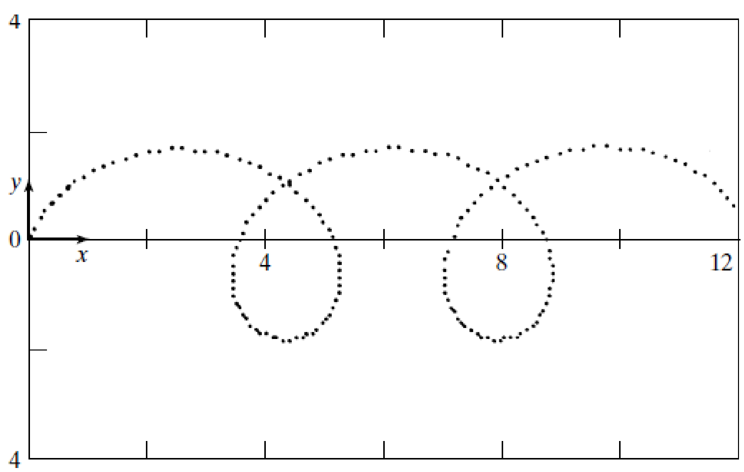


FIGURE 1.41

An example of tracing the path of an electron in crossed electric and magnetic fields.

D1.21. A magnetic field $\mathbf{B} = (B_0/3)(\mathbf{a}_x + 2\mathbf{a}_y - 2\mathbf{a}_z)$ exists at a point. For each of the following velocities of a test charge q, find the electric field $\mathbf{E}$ at that point for which the acceleration experienced by the test charge is zero: **(a)** $v_0(\mathbf{a}_x - \mathbf{a}_y + \mathbf{a}_z)$; **(b)** $v_0(2\mathbf{a}_x + \mathbf{a}_y + 2\mathbf{a}_z)$; and **(c)** v_0 along the line $y = -z = 2x$.

Ans. **(a)** $-v_0B_0(\mathbf{a}_y + \mathbf{a}_z)$; **(b)** $v_0B_0(2\mathbf{a}_x - 2\mathbf{a}_y - \mathbf{a}_z)$; **(c) 0.**

D1.22. In a region of uniform electric and magnetic fields $\mathbf{E} = E_0\mathbf{a}_y$ and $\mathbf{B} = B_0\mathbf{a}_z$, respectively, a test charge q of mass m moves in the manner

$$x = \frac{E_0}{\omega_c B_0}(\omega_c t - \sin \omega_c t)$$

$$y = \frac{E_0}{\omega_c B_0}(1 - \cos \omega_c t)$$

$$z = 0$$

where $\omega_c = qB_0/m$. Find the forces acting on the test charge for the following values of t: **(a)** $t = 0$; **(b)** $t = \pi/2\omega_c$; and **(c)** $t = \pi/\omega_c$.

Ans. **(a)** $qE_0\mathbf{a}_y$; **(b)** $qE_0\mathbf{a}_x$; **(c)** $-qE_0\mathbf{a}_y$.