Session 6 :

## FARADAY'S LAW

In mathematical form, Faraday's law is given by

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

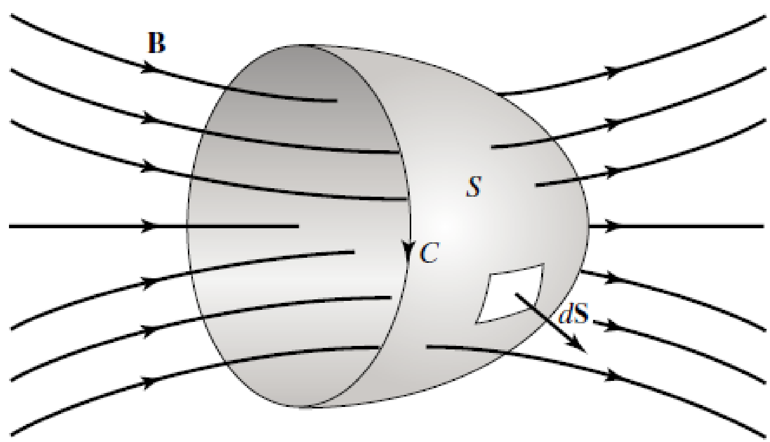



FIGURE 2.10
For illustrating Faraday's law.

1- Right Hand Screw Rule
2- Any Surface S bounded by C
3- The closed Path need not represent a loop of wire
4- Lenz Law : the minus sign on the right hand
5- N when more than one turn

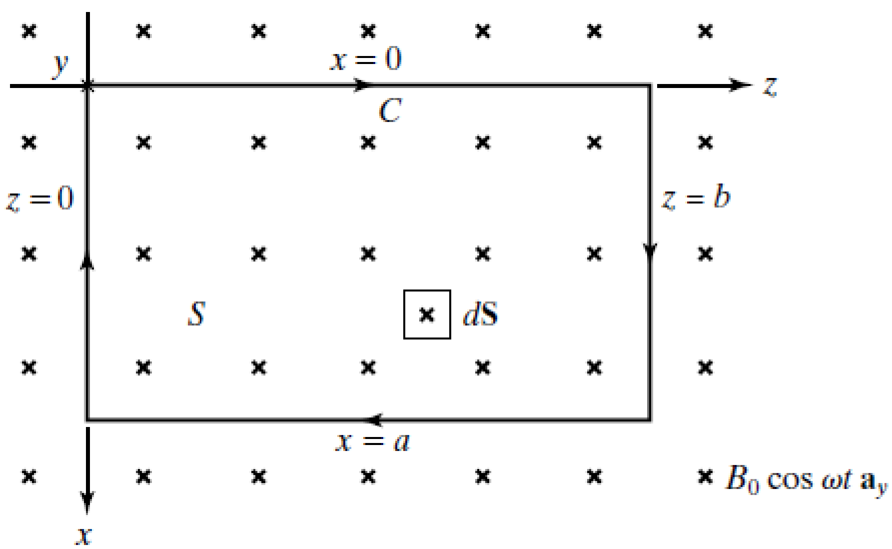
$$\text{emf} = -N\frac{d\psi}{dt}$$

### Example 2.3 Induced emf around a rectangular loop in a time-varying magnetic field

$$\mathbf{B} = B_0 \cos \omega t \, \mathbf{a}_y$$

$$\begin{aligned} \psi &= \int_S \mathbf{B} \cdot d\mathbf{S} = \int_{z=0}^{b} \int_{x=0}^{a} B_0 \cos \omega t \, \mathbf{a}_y \cdot dx \, dz \, \mathbf{a}_y \\ &= B_0 \cos \omega t \int_{z=0}^{b} \int_{x=0}^{a} dx \, dz = ab B_0 \cos \omega t \end{aligned}$$

$$\begin{aligned} \oint_C \mathbf{E} \cdot d\mathbf{l} &= -\frac{d}{dt} \int_S \mathbf{B} \cdot d\mathbf{S} \\ &= -\frac{d}{dt}[ab B_0 \cos \omega t] = ab B_0 \omega \sin \omega t \end{aligned}$$



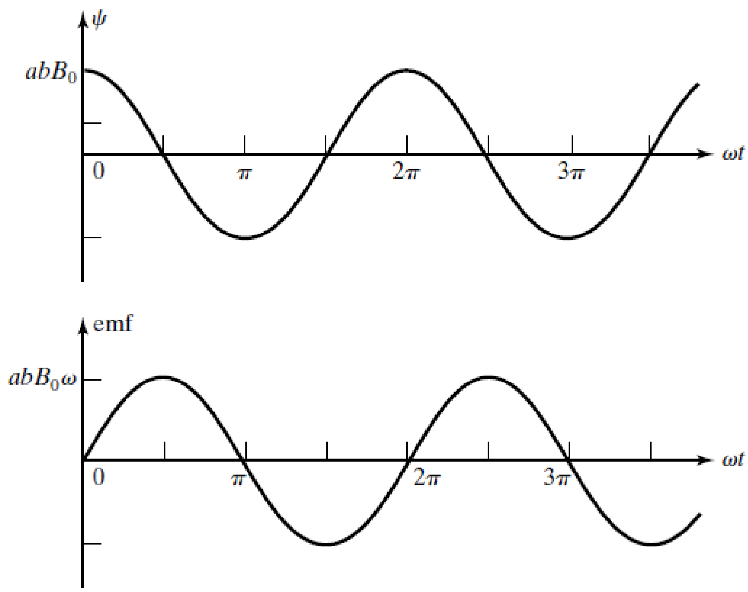



FIGURE 2.15

Time variations of magnetic flux $\psi$ enclosed by the loop of Fig. 2.14 and the resulting induced emf around the loop.

## Example 2.4 Induced emf around an expanding loop in a uniform static magnetic field

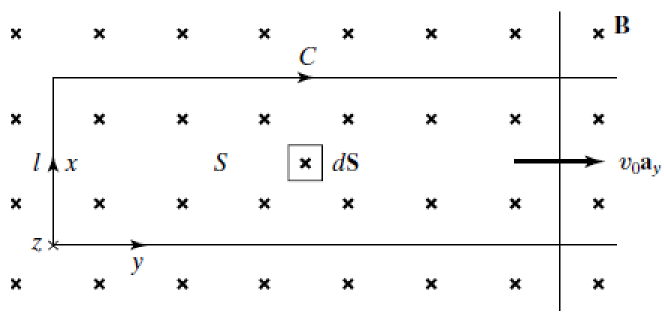



FIGURE 2.16

Rectangular loop of wire with a movable side situated in a uniform magnetic field.

$$\mathbf{B} = B_0\mathbf{a}_z.$$

$$\begin{aligned}\int_S \mathbf{B}\cdot d\mathbf{S} &= \int_S B_0\mathbf{a}_z \cdot dx\,dy\,\mathbf{a}_z \\ &= \int_{x=0}^{l}\int_{y=0}^{y_0+v_0t} B_0\,dx\,dy \\ &= B_0 l(y_0 + v_0 t)\end{aligned}$$

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

$$= -\frac{d}{dt}[B_0 l(y_0 + v_0 t)]$$

$$= -B_0 l v_0$$

*Principle of loop antenna*

That the arrangement considered in Example 2.3 illustrates the principle of a loop antenna can be seen by noting that if the loop $C$ were in the $xy$-plane or in the $yz$-plane, no emf would be induced in it since the magnetic flux density is then parallel to the plane of the loop and no flux is enclosed by the loop. In fact, for any arbitrary orientation of the loop, only that component of **B** normal to the plane of the loop contributes to the magnetic flux enclosed by the loop and, hence, to the emf induced in the loop. Thus, for a given magnetic field, the voltage induced in the loop varies as the orientation of the loop is changed, with the maximum occurring when the loop is in the plane perpendicular to the magnetic field. Pocket AM radios generally contain a type of loop antenna consisting of many turns of wire wound around a bar of magnetic material, and TV receivers generally employ a single-turn circular loop for UHF channels. Thus, for maximum signals to be received, the AM radios and the TV loop antennas need to be oriented appropriately. Another point of interest evi-

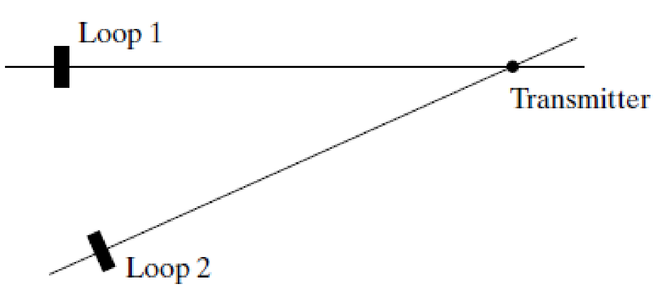



**FIGURE 2.17**

Top view of an arrangement consisting of two loop antennas for locating a transmitter of radio waves.

**D2.6.** A square loop lies in the $xy$-plane forming the closed path $C$ connecting the points $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$, $(0, 1, 0)$, and $(0, 0, 0)$, in that order. A magnetic field $\mathbf{B}$ exists in the region. From considerations of Lenz's law, determine whether the induced emf around the closed path $C$ at $t = 0$ is positive, negative, or zero for each of the following magnetic fields, where $B_0$ is a positive constant: **(a)** $\mathbf{B} = B_0 t\mathbf{a}_z$; **(b)** $\mathbf{B} = B_0 \cos(2\pi t + 60°)\,\mathbf{a}_z$; and **(c)** $\mathbf{B} = B_0 e^{-t^2}\mathbf{a}_z$.

*Ans.* **(a)** negative; **(b)** positive; **(c)** zero.

**D2.7.** For $\mathbf{B} = B_0 \cos \omega t\, \mathbf{a}_z$ Wb/m$^2$, find the induced emf around the following closed paths: **(a)** the closed path comprising the straight lines successively connecting the points $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$, $(0, 1, 0)$, $(0, 0, 0.001)$, and $(0, 0, 0)$; **(b)** the closed path comprising the straight lines successively connecting the points $(0, 0, 0)$, $(1, 0, 0)$, $(1, 1, 0)$, $(0, 1, 0)$, $(0, 0, 0.001)$, $(1, 0, 0.001)$, $(1, 1, 0.001)$, $(0, 1, 0.001)$, $(0, 0, 0.002)$, and $(0, 0, 0)$ with a slight kink in the straight line at the point $(0, 0, 0.001)$ to avoid touching the point; and **(c)** the closed path comprising the helical path $r = 1/\sqrt{\pi}$, $\phi = 1000\pi z$ from $(1/\sqrt{\pi}, 0, 0)$ to $(1/\sqrt{\pi}, 0, 0.01)$ and the straight-line path from $(1/\sqrt{\pi}, 0, 0.01)$ to $(1/\sqrt{\pi}, 0, 0)$ with slight kinks to avoid touching the helical path.

*Ans.* **(a)** $\omega B_0 \sin \omega t$ V; **(b)** $2\omega B_0 \sin \omega t$ V; **(c)** $5\omega B_0 \sin \omega t$ V.

## AMPÈRE'S CIRCUITAL LAW

$$\oint_C \frac{\mathbf{B}}{\mu_0} \cdot d\mathbf{l} = [I_c]_S + \frac{d}{dt}\int_S \varepsilon_0 \mathbf{E} \cdot d\mathbf{S}$$

$$[I_c]_S = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S} + \frac{d}{dt}\int_S \mathbf{D} \cdot d\mathbf{S}$$

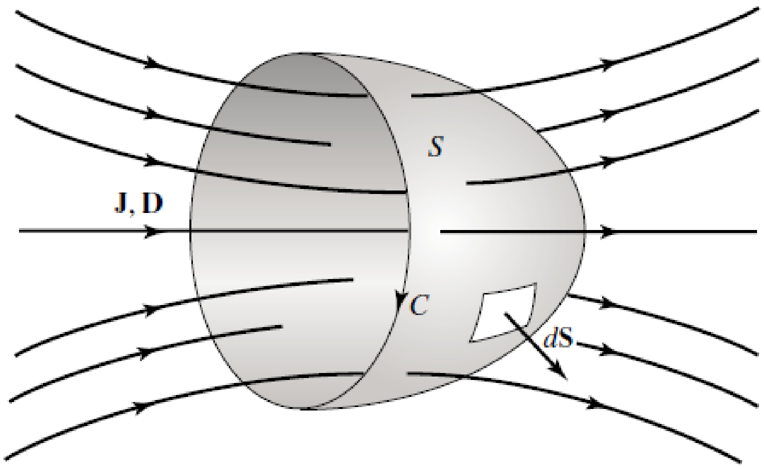



FIGURE 2.22

For illustrating Ampère's circuital law.

1- Right Hand screw Rule
2- Any surface bounded by C