Session 7

Applying Ampère's circuital law to C_1 and S_1 and noting that $d\mathbf{S}_1$ is chosen in accordance with the R.H.S. rule, we have

$$\oint_{C_1} \mathbf{H} \cdot d\mathbf{l} = \int_{S_1} \mathbf{J} \cdot d\mathbf{S}_1 + \frac{d}{dt}\int_{S_1} \mathbf{D} \cdot d\mathbf{S}_1 \qquad (2.18a)$$

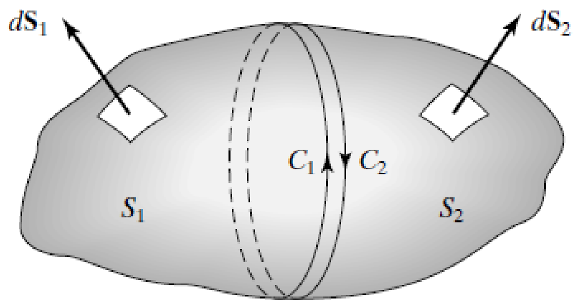


FIGURE 2.23

Two closed paths C_1 and C_2 touching each other and bounding the surfaces S_1 and S_2, respectively, which together form a closed surface.

$$\oint_{C_2} \mathbf{H} \cdot d\mathbf{l} = \int_{S_1} \mathbf{J} \cdot d\mathbf{S}_2 + \frac{d}{dt}\int_{S_2} \mathbf{D} \cdot d\mathbf{S}_2$$

$$0 = \oint_{S_1+S_2} \mathbf{J} \cdot d\mathbf{S} + \frac{d}{dt}\oint_{S_1+S_2} \mathbf{D} \cdot d\mathbf{S}$$

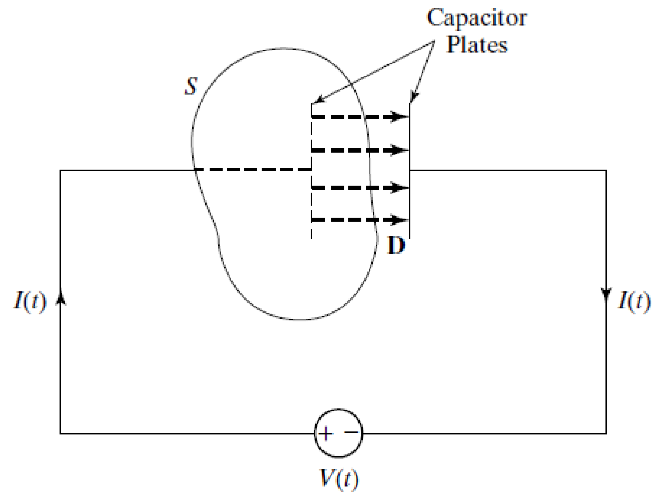
$$\oint_{S} \mathbf{J} \cdot d\mathbf{S} + \frac{d}{dt}\oint_{S} \mathbf{D} \cdot d\mathbf{S} = 0$$

or

$$\boxed{\frac{d}{dt}\oint_{S} \mathbf{D} \cdot d\mathbf{S} = -\oint_{S} \mathbf{J} \cdot d\mathbf{S}} \qquad (2.20)$$

Thus, the displacement current emanating from a closed surface is equal to the current due to charges flowing into the volume bounded by that closed surface.

Capacitor circuit



Radiation from antenna

$$\oint_C \mathbf{E}\cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{S}$$

$$\oint_C \mathbf{H}\cdot d\mathbf{l} = \int_S \mathbf{J}\cdot d\mathbf{S} + \frac{d}{dt}\int_S \mathbf{D}\cdot d\mathbf{S}$$

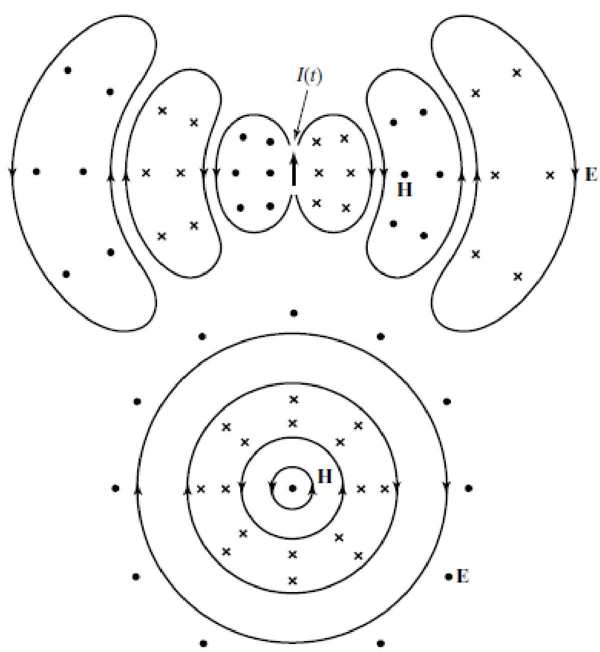


FIGURE 2.25

Two views of a simplified depiction of electromagnetic wave radiation from a piece of wire carrying a time-varying current.

GAUSS' LAWS

Gauss' law for the electric field states that *the displacement flux emanating from a closed surface S is equal to the charge contained within the volume V bounded by that surface.* This statement, although familiarly known as Gauss' law, has its origin in experiments conducted by Faraday. In mathematical form, it is given by

Gauss' law for the electric field

$$\boxed{\oint_S \mathbf{D} \cdot d\mathbf{S} = [Q]_V} \tag{2.25}$$

$$[Q]_V = \int_V \rho \, dv$$

$$\boxed{\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho \, dv}$$

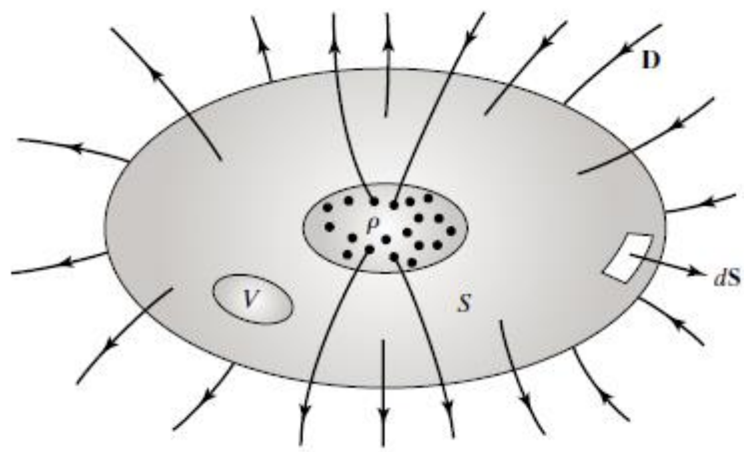


FIGURE 2.26

For illustrating Gauss' law for the electric field.

Gauss' law for the magnetic field

Gauss' law for the magnetic field is analogous to Gauss' law for the electric field and is given by

$$\boxed{\oint_S \mathbf{B} \cdot d\mathbf{S} = 0} \tag{2.28}$$

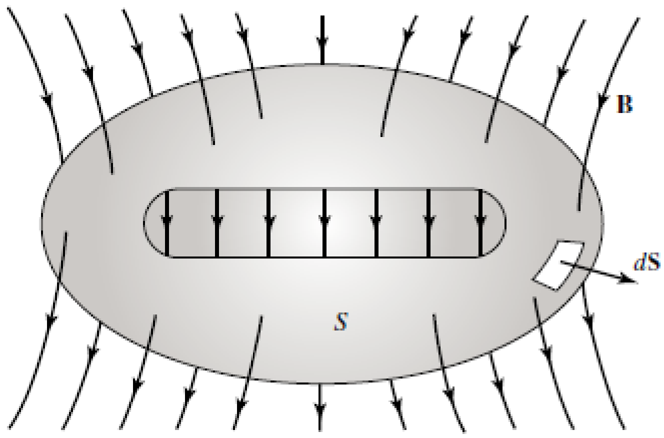


FIGURE 2.27

For illustrating Gauss' law for the magnetic field.

THE LAW OF CONSERVATION OF CHARGE

$$\boxed{\oint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{d}{dt}\int_V \rho\, dv}$$

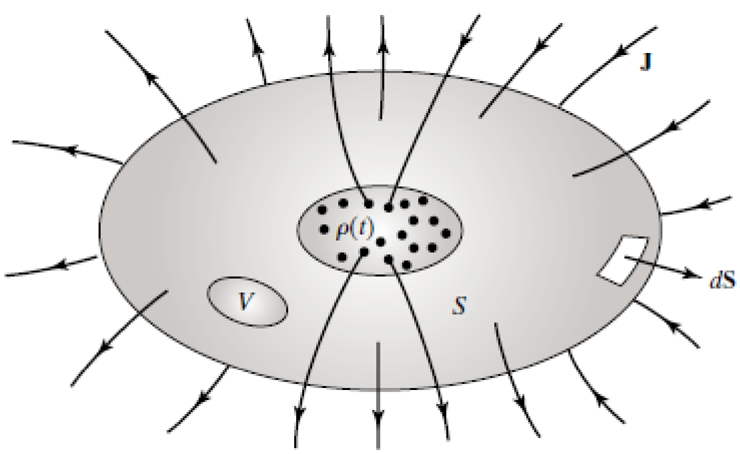


FIGURE 2.29

For illustrating the law of conservation of charge.

$$\frac{d}{dt}\oint_S \mathbf{D}\cdot d\mathbf{S} = \frac{d}{dt}\int_V \rho\, dv$$

$$\frac{d}{dt}\left(\oint_S \mathbf{D}\cdot d\mathbf{S} - \int_V \rho\, dv\right) = 0$$

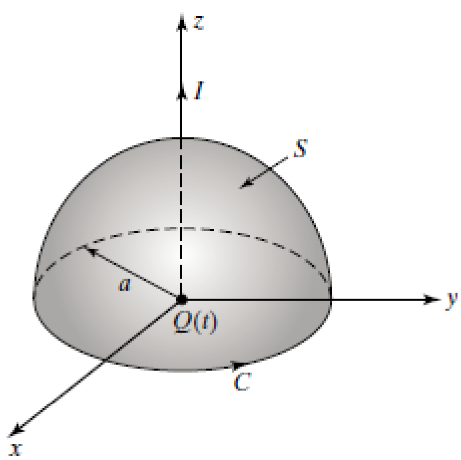
==➔

$$\oint_S \mathbf{D}\cdot d\mathbf{S} = \int_V \rho\, dv$$

$$\oint_S \mathbf{D}\cdot d\mathbf{S} - \int_V \rho\, dv = \text{constant with time}$$

Example 2.5 Combined application of several of Maxwell's equations in integral form

$$\oint_C \mathbf{H}\cdot d\mathbf{l} = I + \frac{d}{dt}\int_S \mathbf{D}\cdot d\mathbf{S}$$



$$\int_S \mathbf{D}\cdot d\mathbf{S} = \frac{Q}{2}$$

$$I = -\frac{dQ}{dt}$$

$$\begin{aligned}\oint_C \mathbf{H}\cdot d\mathbf{l} &= I + \frac{d}{dt}\left(\frac{Q}{2}\right)\\ &= I + \frac{1}{2}\frac{dQ}{dt}\\ &= I + \frac{1}{2}(-I)\\ &= \frac{I}{2}\end{aligned}$$