Session 8

## APPLICATION TO STATIC FIELDS

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{d}{dt}\int_S \mathbf{B} \cdot d\mathbf{S}$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S} + \frac{d}{dt}\int_S \mathbf{D} \cdot d\mathbf{S}$$

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho \, dv$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

حالت استاتیک

(تغییرات با زمان نداریم)

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = 0$$

$$\oint_C \mathbf{H} \cdot d\mathbf{l} = \int_S \mathbf{J} \cdot d\mathbf{S}$$

$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho \, dv$$

$$\oint_S \mathbf{B} \cdot d\mathbf{S} = 0$$

$$\oint_S \mathbf{J} \cdot d\mathbf{S} = -\frac{d}{dt}\int_V \rho \, dv \quad \rightarrow \quad \oint_S \mathbf{J} \cdot d\mathbf{S} = 0$$

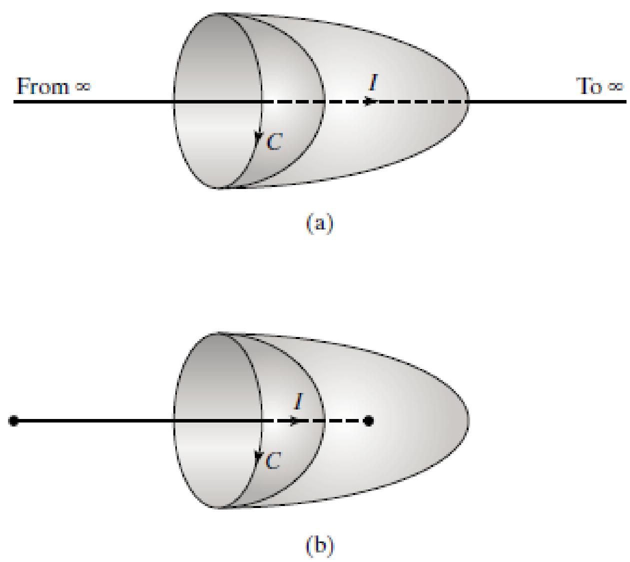
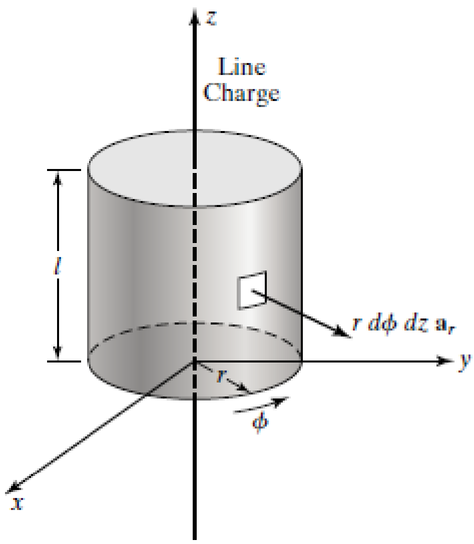



FIGURE 2.31

For illustrating that the current enclosed by a closed path $C$ is uniquely given in (a) but not in (b).

### Example 2.6 Electric field due to an infinitely long line charge using Gauss' law




$$\oint_S \mathbf{D} \cdot d\mathbf{S} = \rho_{L0} l$$

**(a)** In view of the uniform charge density, the entire line charge can be thought of as the superposition of pairs of equal point charges located at equal distances above and below any given point on the $z$-axis. Hence the field due to the entire line charge has only a radial component independent of $\phi$ and $z$.

**(b)** In view of (a), the contribution to the closed surface integral from the top and bottom surfaces of the cylindrical box is zero.

Thus, we have

$$\mathbf{D} = D_r(r)\mathbf{a}_r$$

$$\begin{aligned}\oint_S \mathbf{D} \cdot d\mathbf{S} &= \int_{\phi=0}^{2\pi} \int_{z=0}^{l} D_r(r)\mathbf{a}_r \cdot r\, d\phi\, dz\, \mathbf{a}_r \\ &= 2\pi r l D_r(r)\end{aligned}$$

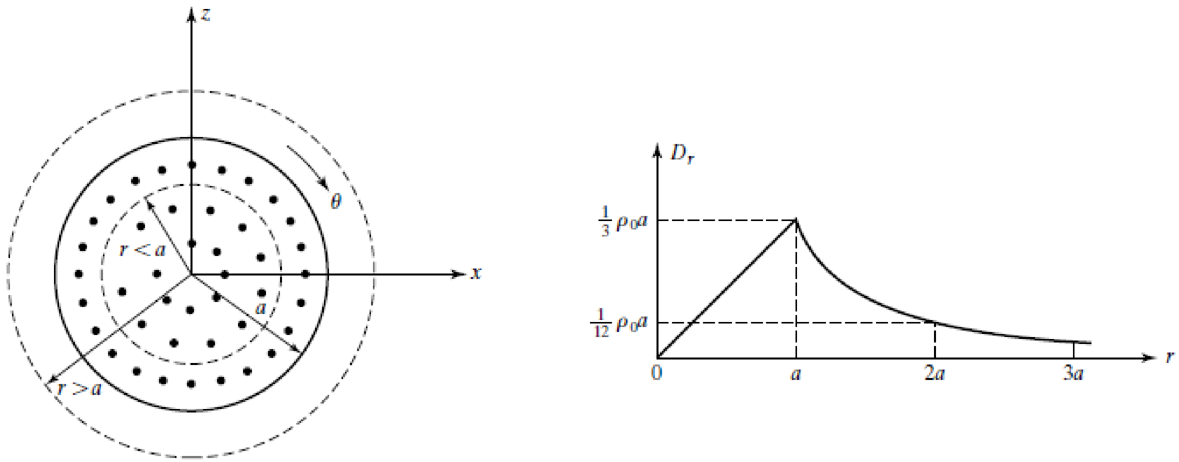
Comparing (2.39) and (2.40), we obtain

$$\begin{aligned}2\pi r l D_r(r) &= \rho_{L0} l \\ D_r(r) &= \frac{\rho_{L0}}{2\pi r}\end{aligned}$$

$$\boxed{\mathbf{D} = \frac{\rho_{L0}}{2\pi r}\mathbf{a}_r}$$

The field varies inversely with the radial distance away from the line charge.

### Example 2.7 Electric field due to a spherical volume charge using Gauss' law




$$\mathbf{D} = D_r(r)\mathbf{a}_r$$

Choosing, then, a spherical surface of radius $r$ centered at the origin, we obtain

$$\begin{aligned}\oint_S \mathbf{D}\cdot d\mathbf{S} &= \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} D_r(r)\mathbf{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi \, \mathbf{a}_r \\ &= 4\pi r^2 D_r(r)\end{aligned} \tag{2.42}$$

Noting that the charge exists only for $r < a$, and with uniform density, we obtain the charge enclosed by the spherical surface to be

$$\int_V \rho \, dv = \begin{cases} \frac{4}{3}\pi r^3 \rho_0 & \text{for} \quad r \le a \\ \frac{4}{3}\pi a^3 \rho_0 & \text{for} \quad r \ge a \end{cases} \tag{2.43}$$

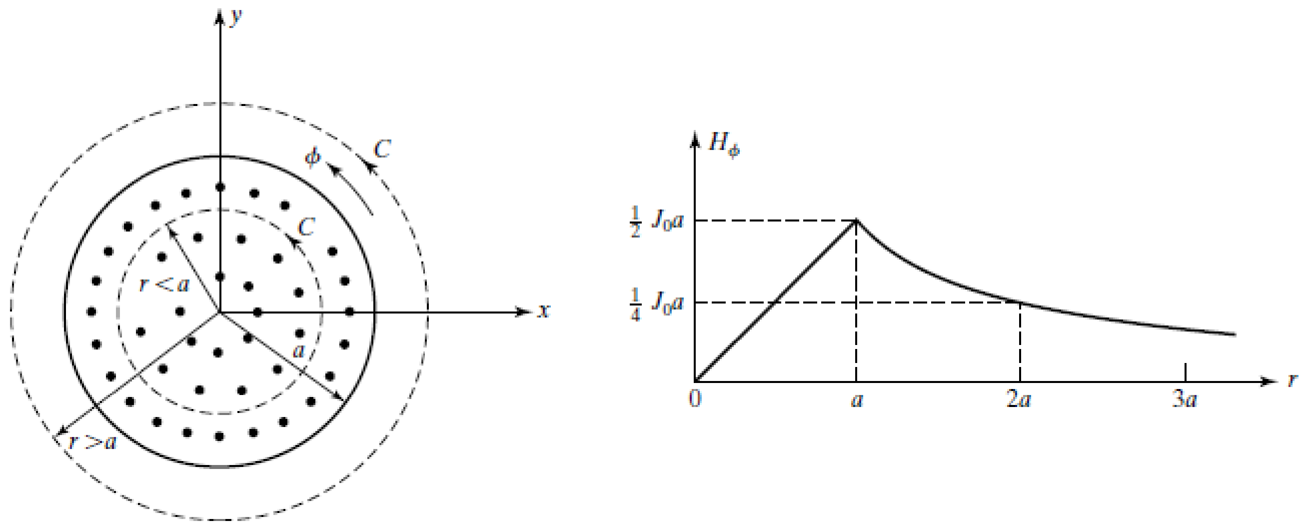
Substituting (2.42) and (2.43) into (2.37c), we get

$$4\pi r^2 D_r(r) = \begin{cases} \frac{4}{3}\pi r^3 \rho_0 & \text{for} \quad r \le a \\ \frac{4}{3}\pi a^3 \rho_0 & \text{for} \quad r \ge a \end{cases}$$

$$D_r(r) = \begin{cases} \dfrac{\rho_0 r}{3} & \text{for} \quad r \le a \\ \dfrac{\rho_0 a^3}{3r^2} & \text{for} \quad r \ge a \end{cases}$$

$$\boxed{\mathbf{D} = \begin{cases} \dfrac{\rho_0 r}{3}\mathbf{a}_r & \text{for} \quad r \le a \\ \dfrac{\rho_0 a^3}{3r^2}\mathbf{a}_r & \text{for} \quad r \ge a \end{cases}} \tag{2.44}$$

### Example 2.8 Magnetic field due to cylindrical wire of current using Ampere's circuital law




$$\mathbf{H} = H_\phi(r)\mathbf{a}_\phi$$

Choosing, then, a circular closed path $C$ of radius $r$ lying in the $xy$-plane and centered at the origin, we obtain

$$\begin{aligned}\oint_C \mathbf{H}\cdot d\mathbf{l} &= \int_{\phi=0}^{2\pi} H_\phi(r)\mathbf{a}_\phi\cdot r\,d\phi\,\mathbf{a}_\phi \\ &= 2\pi r H_\phi(r)\end{aligned} \tag{2.45}$$

Considering the plane surface bounded by $C$, and noting that the current exists only for $r < a$, we obtain the current enclosed by the closed path to be

$$\begin{aligned}\int_S \mathbf{J}\cdot d\mathbf{S} &= \begin{cases}\displaystyle\int_{r=0}^{r}\int_{\phi=0}^{2\pi} J_0\mathbf{a}_z\cdot r\,dr\,d\phi\,\mathbf{a}_z & \text{for } r \le a \\ \displaystyle\int_{r=0}^{a}\int_{\phi=0}^{2\pi} J_0\mathbf{a}_z\cdot r\,dr\,d\phi\,\mathbf{a}_z & \text{for } r \ge a\end{cases} \\ &= \begin{cases}J_0\pi r^2 & \text{for } r \le a \\ J_0\pi a^2 & \text{for } r \ge a\end{cases}\end{aligned} \tag{2.46}$$

Substituting (2.45) and (2.46) into (2.37b), we get

$$2\pi r H_\phi = \begin{cases}J_0\pi r^2 & \text{for } r \le a \\ J_0\pi a^2 & \text{for } r \ge a\end{cases}$$

$$H_\phi = \begin{cases}\dfrac{J_0 r}{2} & \text{for } r \le a \\ \dfrac{J_0 a^2}{2r} & \text{for } r \ge a\end{cases}$$

*Principle of rotating generator*

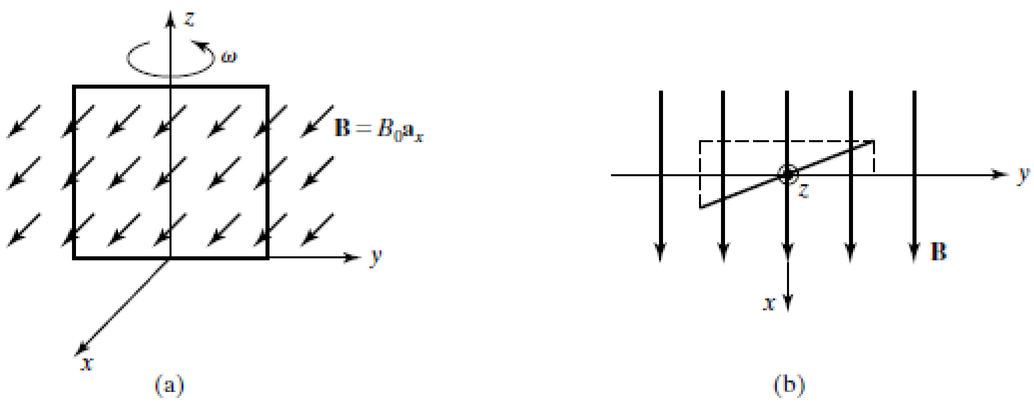



FIGURE 2.18

For illustrating the principle of a rotating generator.

*Magnetic levitation*

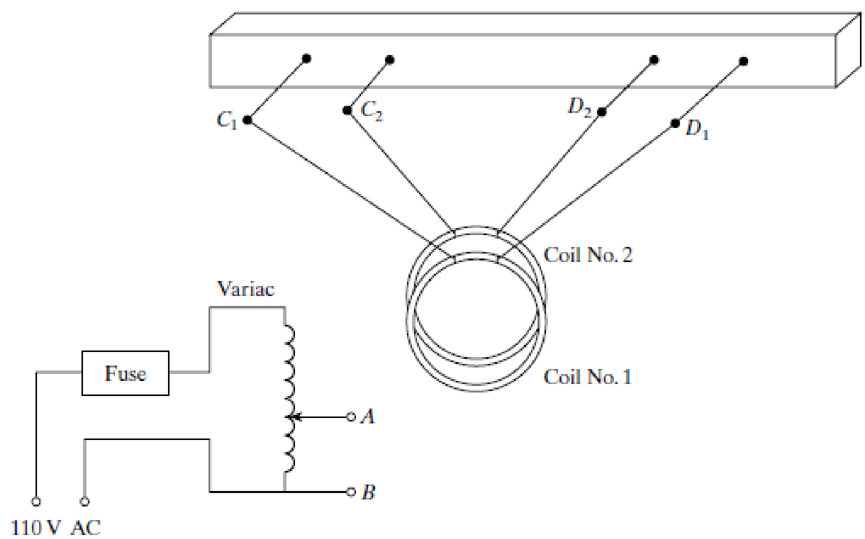



FIGURE 2.19

Experimental setup for demonstration of Ampère's force law, Faraday's law, and the principle of magnetic levitation.

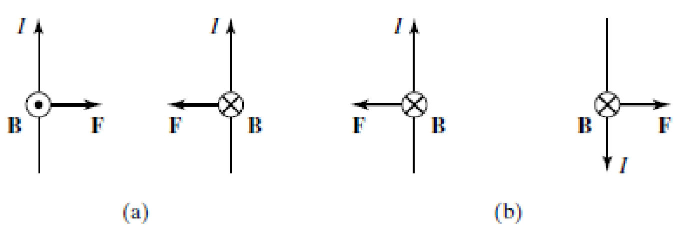



FIGURE 2.20

For explaining (a) force of attraction for currents flowing in the same sense and (b) force of repulsion for currents flowing in opposite senses.

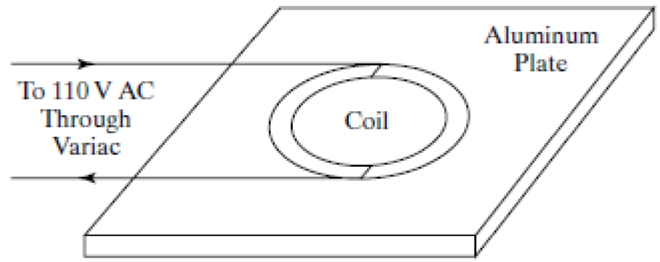



FIGURE 2.21

Setup for demonstrating magnetic levitation.

**D2.13.** Charge is distributed with uniform density $\rho_0$ C/m$^3$ inside a regular solid of edges $a$. Find the displacement flux emanating from one side of the solid for each of the following shapes of the solid: **(a)** tetrahedron; **(b)** cube; and **(c)** octahedron.

*Ans.* **(a)** $0.0295\rho_0 a^3$ C; **(b)** $0.1667\rho_0 a^3$ C; **(c)** $0.0589\rho_0 a^3$ C.

**D2.14.** The cross section of an infinitely long solid wire having the $z$-axis as its axis is a regular polygon of sides $a$. Current flows in the wire with uniform density $J_0\mathbf{a}_z$ A/m$^2$. Find the line integral of $\mathbf{H}$ along one side of the polygon and traversed in the sense of increasing $\phi$ for each of the following shapes of the polygon: **(a)** equilateral triangle; **(b)** square; and **(c)** octagon.

*Ans.* **(a)** $0.1443a^2J_0$ A; **(b)** $0.25a^2J_0$ A; **(c)** $0.6036a^2J_0$ A.