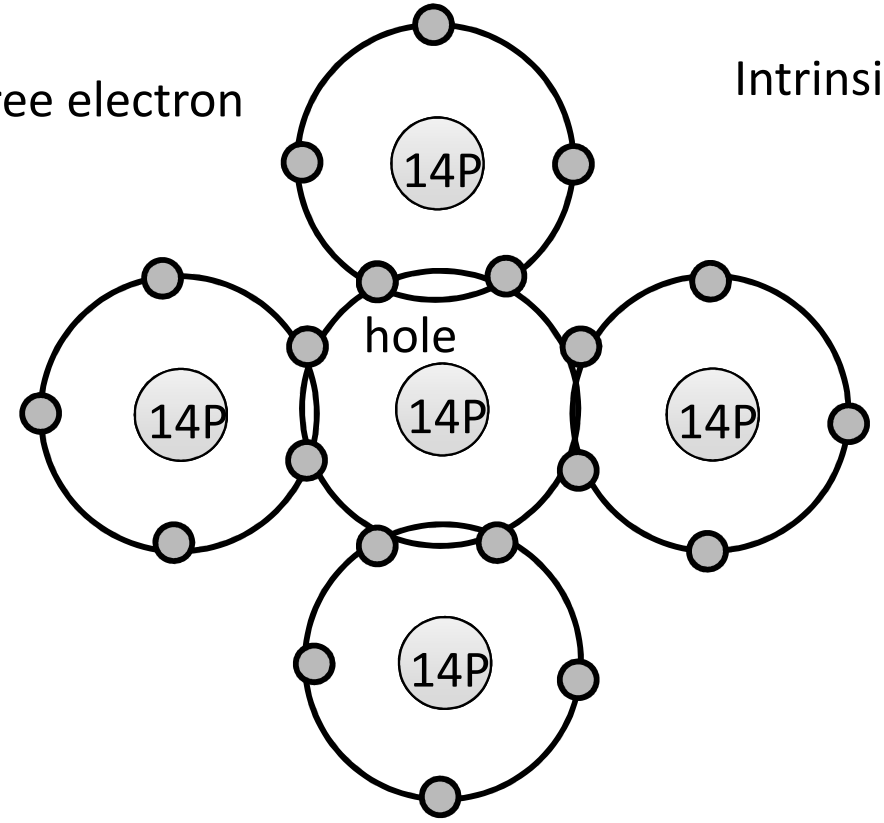
Session 4: Solid State Physics

Dopants and Carrier Concentration

Outline

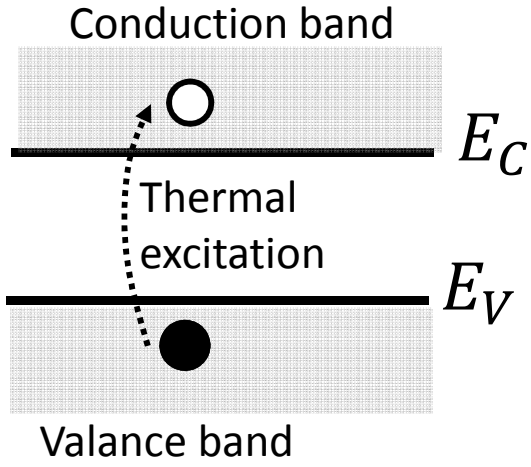
- A
  - B
  - C
  - D
  - E
- F
  - G
- H
- I
- J

Intrinsic Material



Intrinsic = pure

electron/hole pair is generated



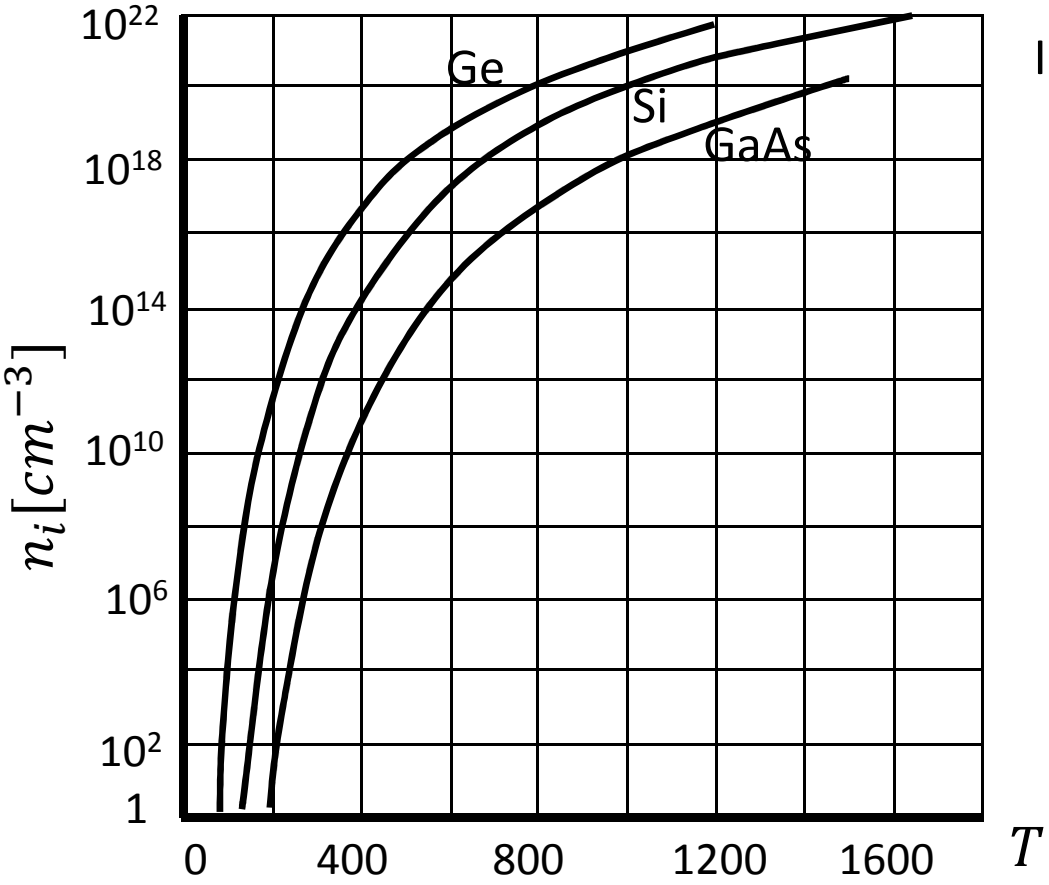
$$n = p = n_i(T)$$

n= # of e^-/cm^3
p= # of h^+/cm^3
n_i= intrinsic carrier concentration

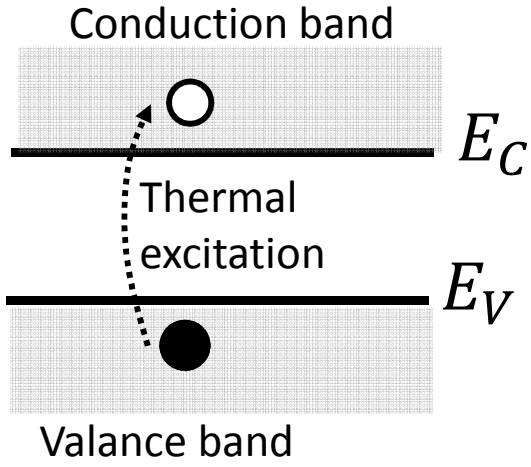
$n_{i-Si}\Big|_{T=300°K} = 1.5 \times 10^{10}\ cm^{-3}$ note that totally there are $2 \times 10^{23} cm^{-3}$ electrons

1 out of 10^{13} bond is broken!

Intrinsic Material



Intrinsic = pure



Energy Band Gap determines the intrinsic carrier concentration. ni EgGe< EgSi< EgGaAs

Doping

Doping means mixing a pure semiconductor with impurities to increase its electrical conductivity

Can be done in two ways:

Increasing the number of electrons by mixing pentavalent elements such as phosphorous, arsenic, antimony (means adding donor impurities)

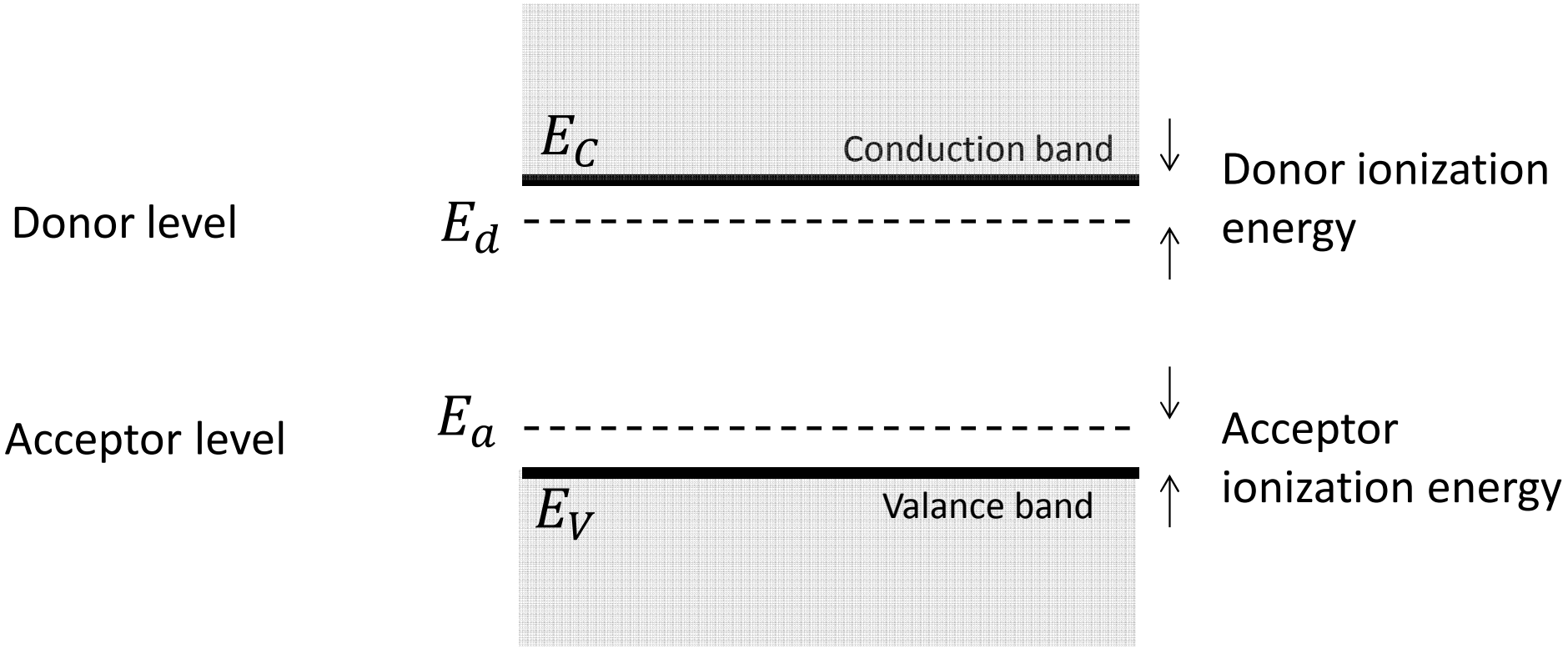
Increasing the number of holes by mixing trivalent elements such as aluminum, boron, gallium (means adding acceptor impurities)

Donors and acceptors are known as dopants.
Dopant ionization energy ~50meV (very low).

Possible dopant deactivation & defect formation

N^{--} or P^{--} :	N_D or $N_A < 10^{14}$ cm^{-3}
N^{-} or P^{-} :	10^{14} $cm^{-3} < N_D$ or $N_A < 10^{16}$ cm^{-3}
N or P :	10^{16} $cm^{-3} < N_D$ or $N_A < 10^{18}$ cm^{-3}
N^{+} or P^{+} :	10^{18} $cm^{-3} < N_D$ or $N_A < 10^{20}$ cm^{-3}
N^{++} or P^{++} :	N_D or $N_A > 10^{20}$ cm^{-3}

Donor and Acceptor in the Band Model



$@T = 300°K$ $KT = 26meV$

Ionization energy of selected donors and acceptors in silicon

	Donors			Acceptors		
Dopant	Sb	P	As	B	Al	In
Ionization Eng Ec-Ed or Ea-Ev (meV)	39	44	54	45	57	160

Donor and Acceptor in the Band Model

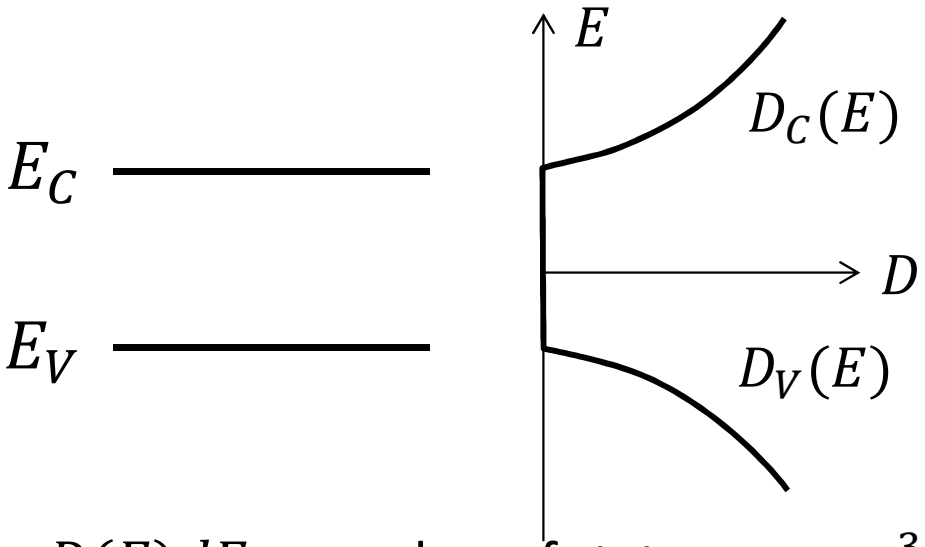
Si

Ionization energies of shallow DONORS(eV) As = 0.054; P = 0.045; Sb = 0.043

Ionization energies of shallow ACCEPTORS(eV) Al = 0.072; B = 0.045; Ga = 0.074; In = 0.157

Ionization energy of various impurities in Ge, Si, and GaAs at 300K.

Density of States



$D(E)dE$ = number of states per cm^3 in the energy range between E and $E + dE$ near the band edges:

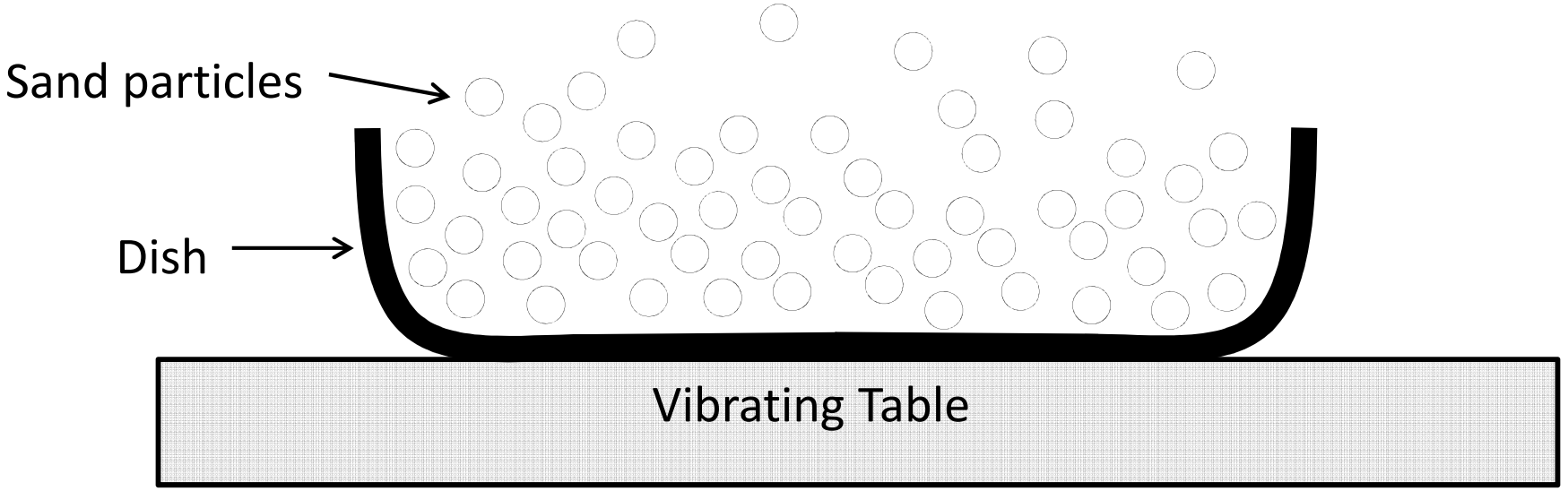
$$D_C(E) \equiv \frac{\text{number of states in } \Delta E}{\Delta E \text{ volume}} \quad \left(\frac{1}{eV\ .cm^3}\right)$$

$$g_C(E) = D_C(E) = \frac{m_n\sqrt{2m_n(E - E_C)}}{\pi^2 h^3}$$

$$g_V(E) = D_V(E) = \frac{m_p\sqrt{2m_p(E_V - E)}}{\pi^2 h^3}$$

Thermal Equilibrium and Fermi Function

An Analogy for Thermal Equilibrium



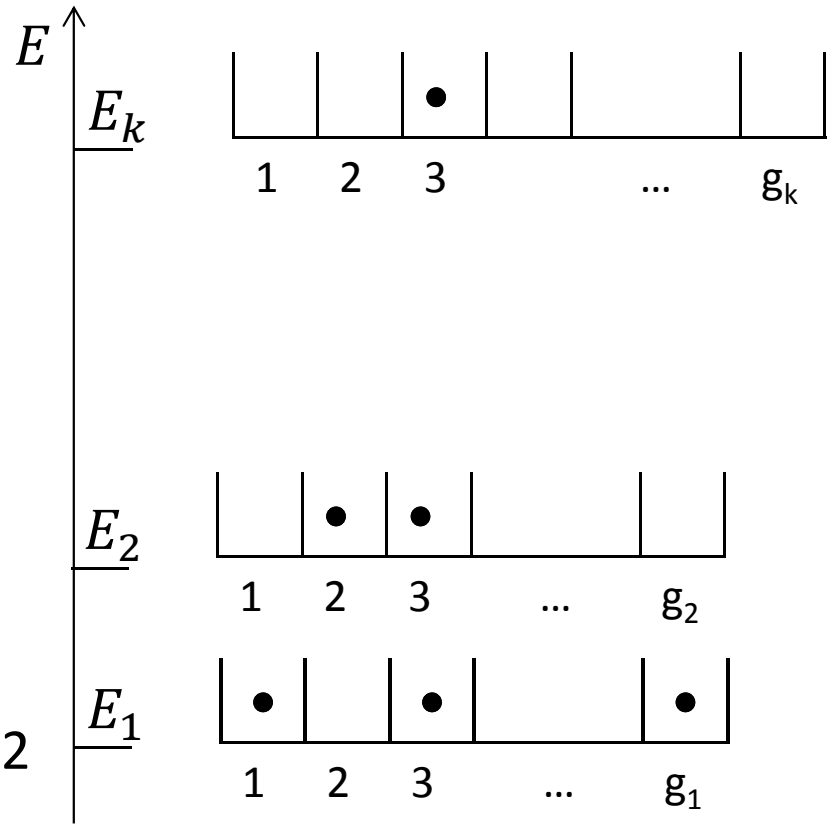
There is a certain probability for the electrons in the conduction band to occupy high-energy states under the agitation of thermal energy.

Probability of a State at E being Occupied

•There are g1 states at E1, g2 states at E2… There are N electrons, which constantly shift among all the states but the average electron energy is fixed at 3kT/2.

•There are many ways to distribute N among n1, n2, n3….and satisfy the 3kT/2 condition.

•The equilibrium distribution is the distribution that maximizes the number of combinations of placing n1 in g1 slots, n2 in g2 slots…. :



$$\frac{n_i}{g_i} = \frac{1}{1 + e^{(E-E_F)/kT}}$$

E_F is a constant determined by the condition $\sum_i n_i = N$

Fermi Function

Particles can be classified into 3 categories:

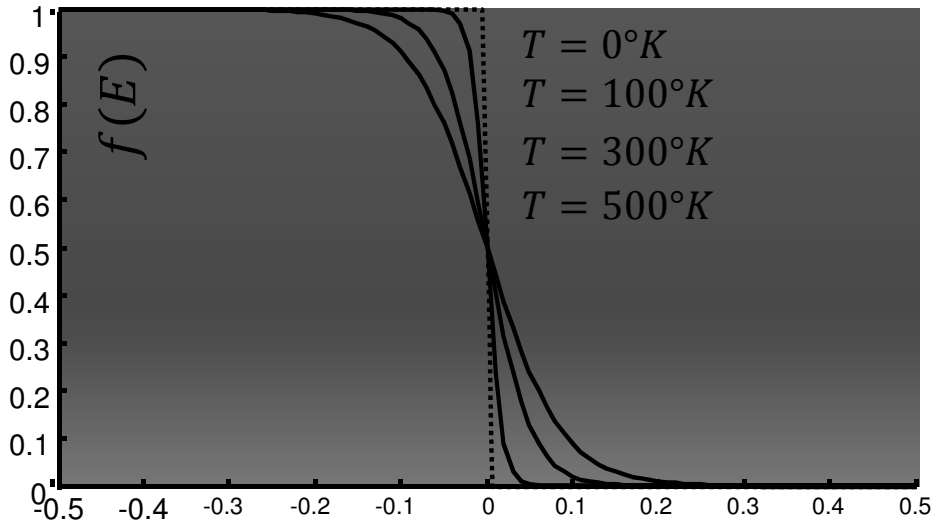
1. Classical particles	(ball)	Maxwell-Boltzmann dist.
2. Bosons	(photons)	Bose-Einstein dist.
3. Fermions(undist + Pauli Exclution)	(electrons)	Fermi-Dirac dist.

Probability that an available state at energy E is occupied:

$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

Ef is called the Fermi energy or the Fermi level.
There is only one Fermi level in a system at equilibrium.

Boltzmann constant,
$k = 1.38 \times 10^{-23}$ J/K
$k = 8.62 \times 10^{-5}$ eV/K



Fermi Function
Probability of Electron Distribution

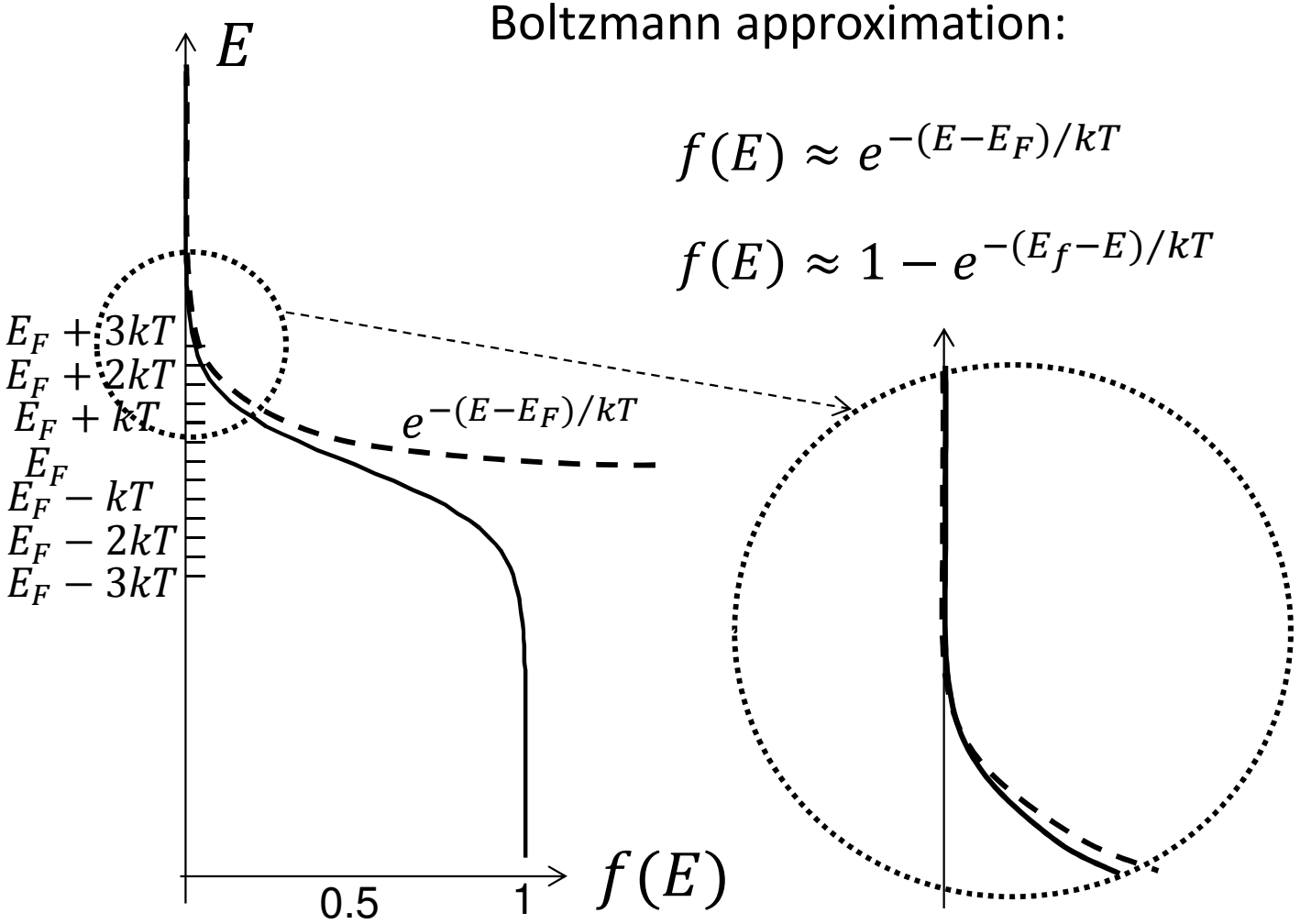
$$f(E) = \frac{1}{1 + e^{(E-E_F)/kT}}$$

Ef is called the Fermi energy or the Fermi level.

If we are 3kT away from the Fermi energy then we might use Boltzmann approximation:

$$f(E) \approx e^{-(E-E_F)/kT} \quad \text{if} \quad E - E_F \gg kT$$

$$f(E) \approx 1 - e^{-(E_f-E)/kT} \quad \text{if} \quad E - E_F \ll -kT$$



$$-\frac{\partial f}{\partial E} \approx \delta(E - E_F)$$

Electron / Hole Concentrations

Derivation of n and p from D(E) and f(E)

Integrate n(E) over all the energies in the conduction band to obtain n

$$n = \int_{E_c}^{\text{Top Of Conduction Band}} f(E)D_C(E)dE = \frac{m_n\sqrt{2m_n}}{\pi^2 h^3}\int_{E_c}^{\infty}\sqrt{E-E_C}\; e^{-(E-E_F)/kT}\,dE$$

$$n = \frac{m_n\sqrt{2m_n}}{\pi^2 h^3}\; e^{-(E_C-E_F)/kT}\int_0^{\infty}\sqrt{\eta}\, e^{-\eta/kT}\,d\eta \qquad (\eta = E - E_C)$$

$$n = N_C\, e^{-(E_C-E_F)/kT} \qquad N_C \equiv 2\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2}$$

Nc is called the effective density of states of the conduction band.

$$p = \int_{\text{Top Of Conduction Band}}^{E_V} [1-f(E)]D_V(E)dE$$

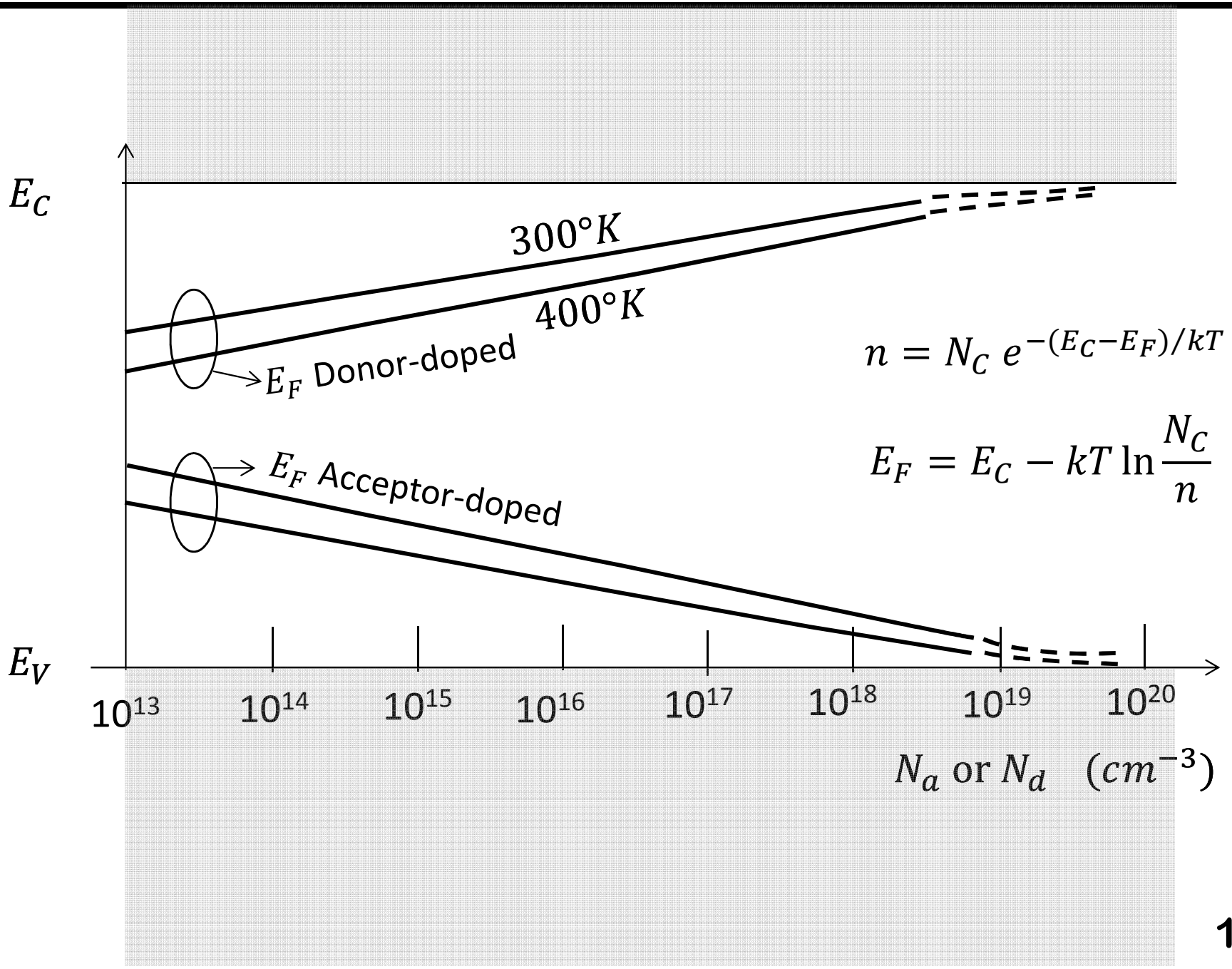
$$p = N_V\, e^{-(E_F-E_V)/kT} \qquad N_V \equiv 2\left[\frac{2\pi m_p kT}{h^2}\right]^{3/2}$$

Nv is called the effective density of states of the valence band.

Closer Ef to Ec the larger n , closer Ef to Ev the larger p

For Si: Nc =2.8×1019 cm-3 , Nv =1.04×1019 cm-3

Fermi Level and Carrier Concentration



$$n = N_C\, e^{-(E_C - E_F)/kT}$$

$$E_F = E_C - kT \ln \frac{N_C}{n}$$

Carrier Concentrations

Multiply $n = N_C\, e^{-(E_C - E_F)/kT}$ and $p = N_V\, e^{-(E_F - E_V)/kT}$

$$np = N_C N_V\, e^{-(E_C - E_V)/kT} = N_C N_V\, e^{-E_G/kT}$$

Law of Mass Action $np = n_i^2$ $\qquad n_i \equiv \sqrt{N_C N_V}\, e^{-E_G/2kT}$

In an intrinsic (undopped) semiconductor, $n = p = n_i$

$$N_C \equiv 2\left[\frac{2\pi m_n kT}{h^2}\right]^{3/2} \qquad N_V \equiv 2\left[\frac{2\pi m_p kT}{h^2}\right]^{3/2} \qquad n_i(T) = \cdots$$

$$E_i \equiv E_F\Big|_{n=p} \rightarrow E_i = \frac{E_C + E_V}{2} + \frac{kT}{2}\ln\frac{N_V}{N_C} = \frac{E_C + E_V}{2} + \frac{3kT}{4}\ln\left(\frac{m_p^*}{m_n^*}\right) \approx \frac{E_C + E_V}{2}$$

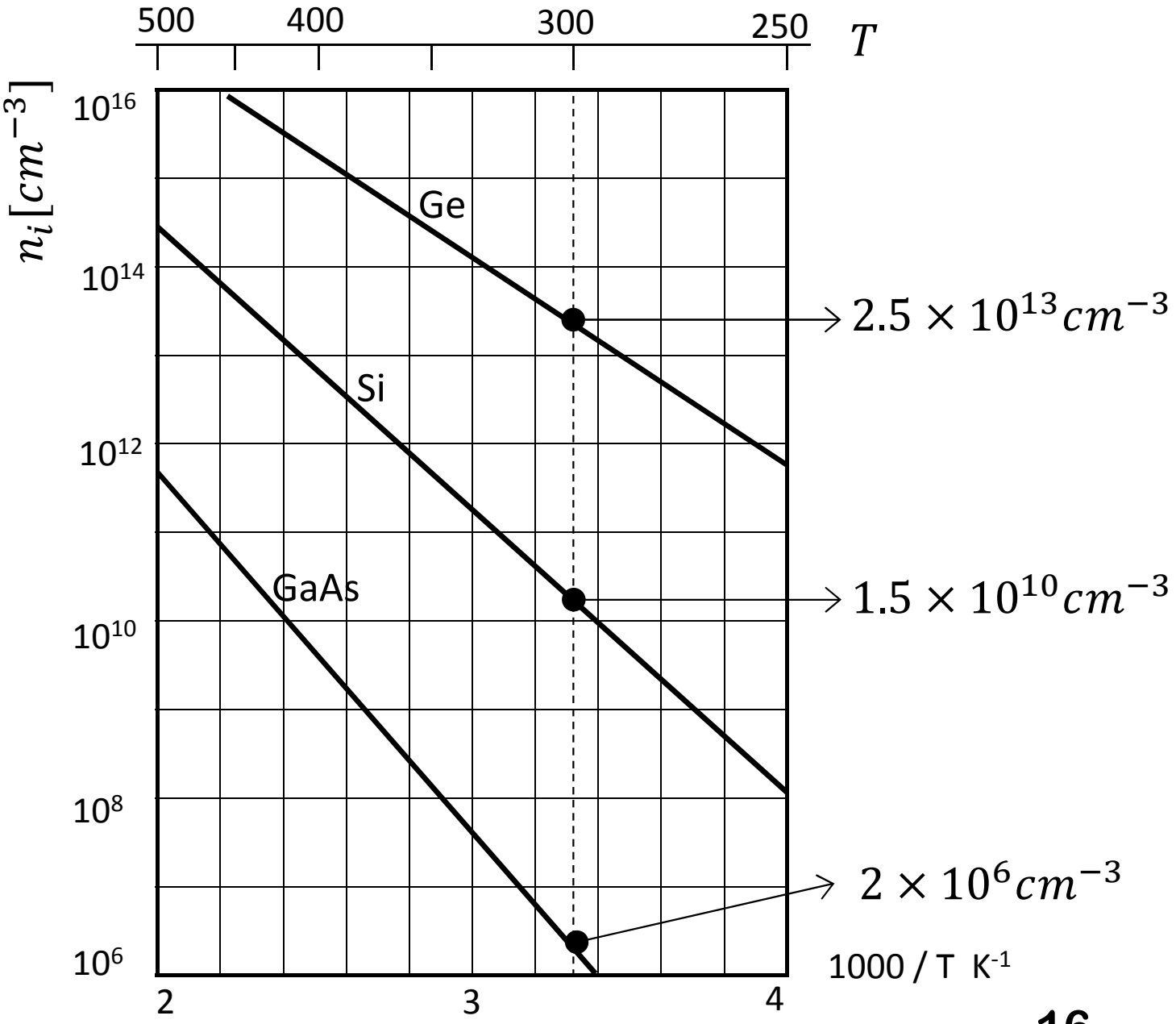
$$\begin{aligned} n &= N_C\, e^{-(E_C - E_F)/kT} \\ p &= N_V\, e^{-(E_F - E_V)/kT} \end{aligned} \xrightarrow{E_i} \begin{cases} n = n_i\, e^{(E_F - E_i)/kT} \\ p = n_i\, e^{(E_i - E_F)/kT} \end{cases}$$

Intrinsic Material

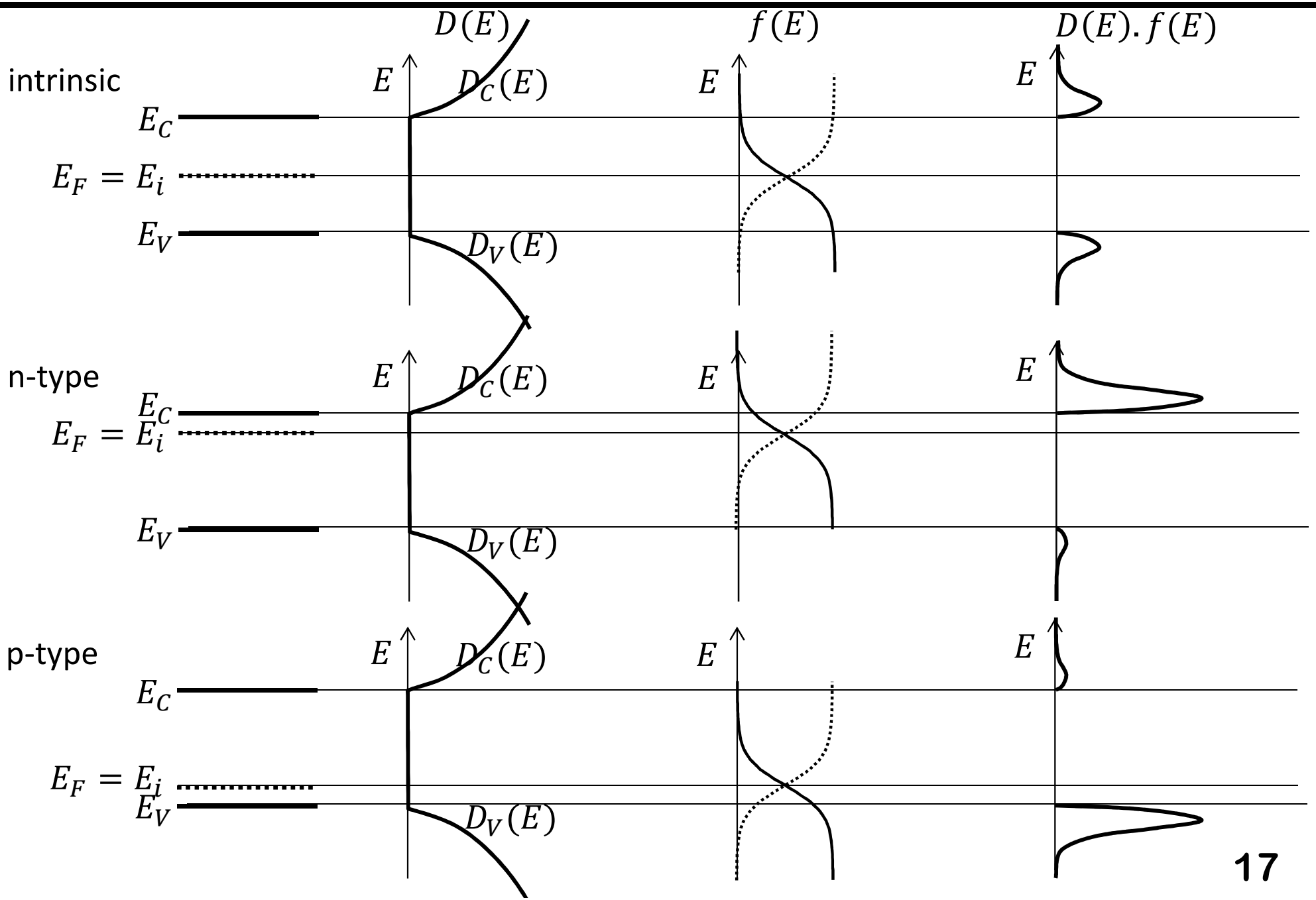
$$n_i \equiv \sqrt{N_C N_V}\, e^{-E_G/2kT}$$

$$n_i = 2\left[\frac{2\pi kT}{h^2}\right]^{3/2}$$

$$\left(m_n^* m_p^*\right)^{3/4}\, e^{-E_G/2kT}$$

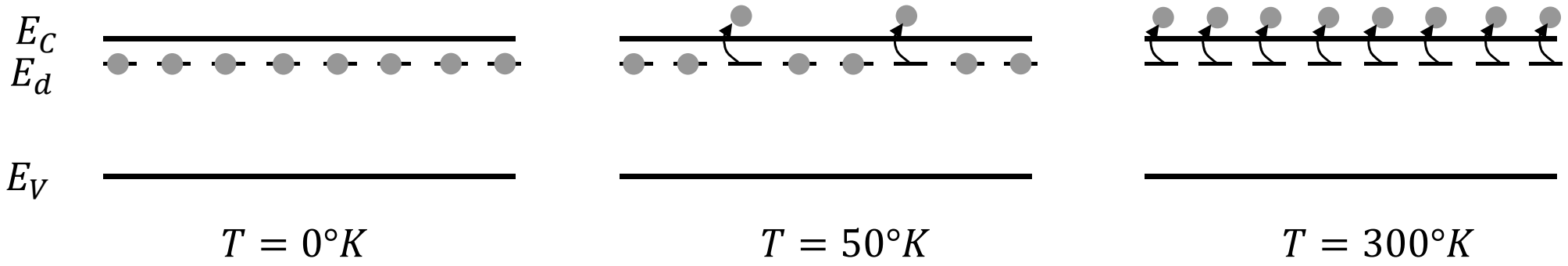


Density of States

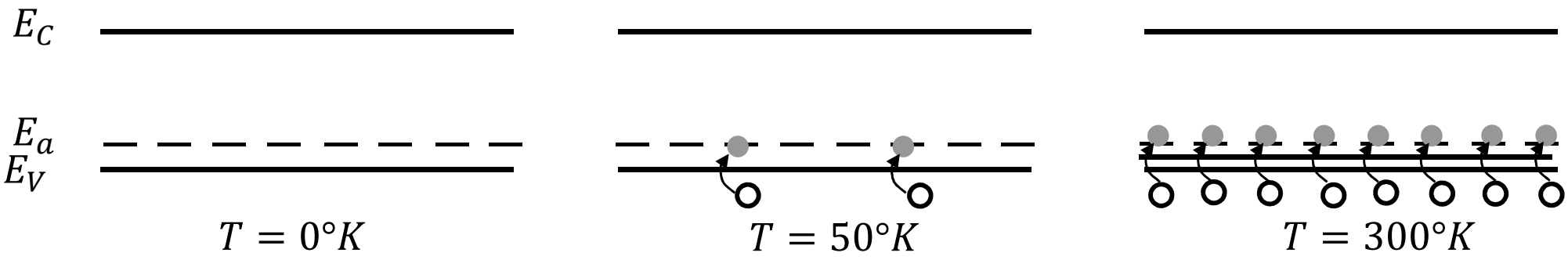


Density of States

Donor



Acceptor

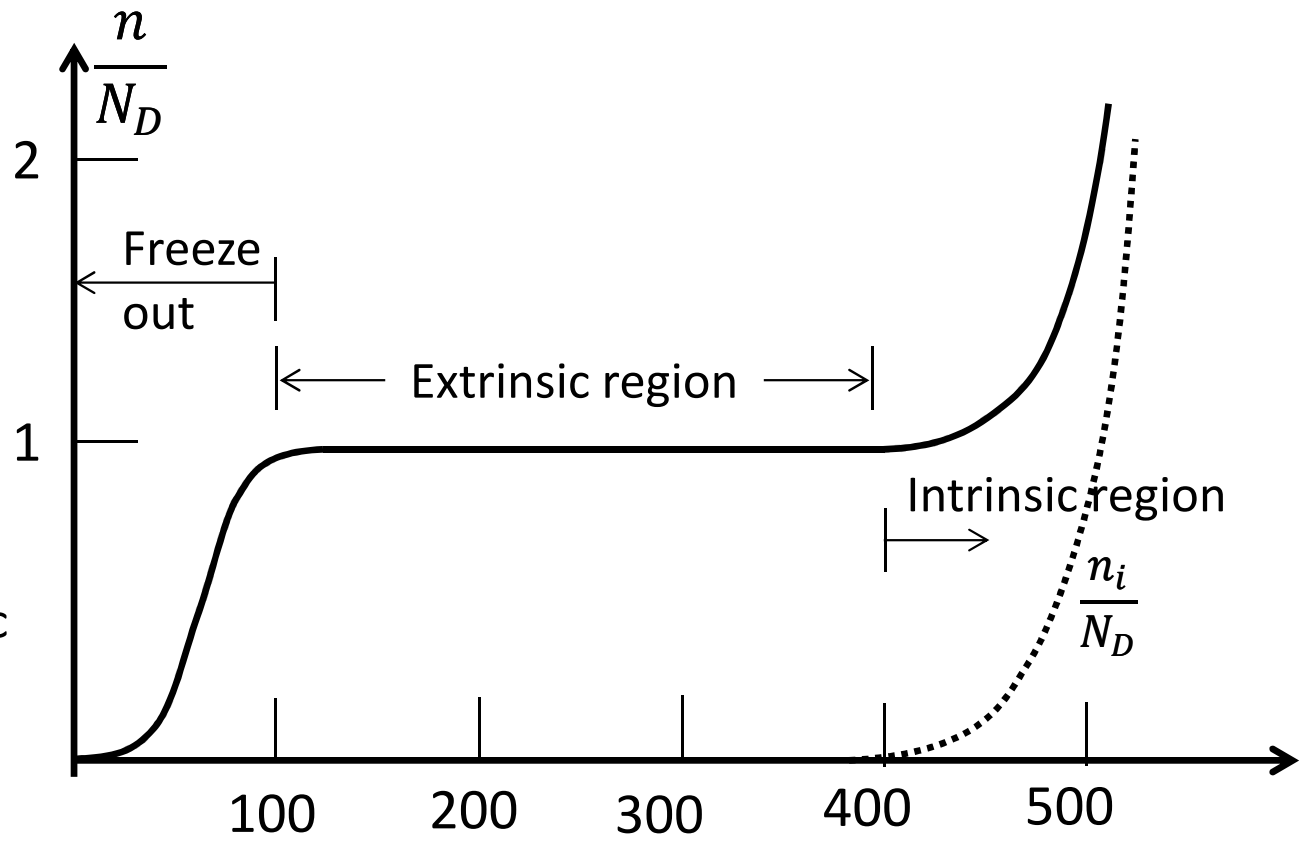


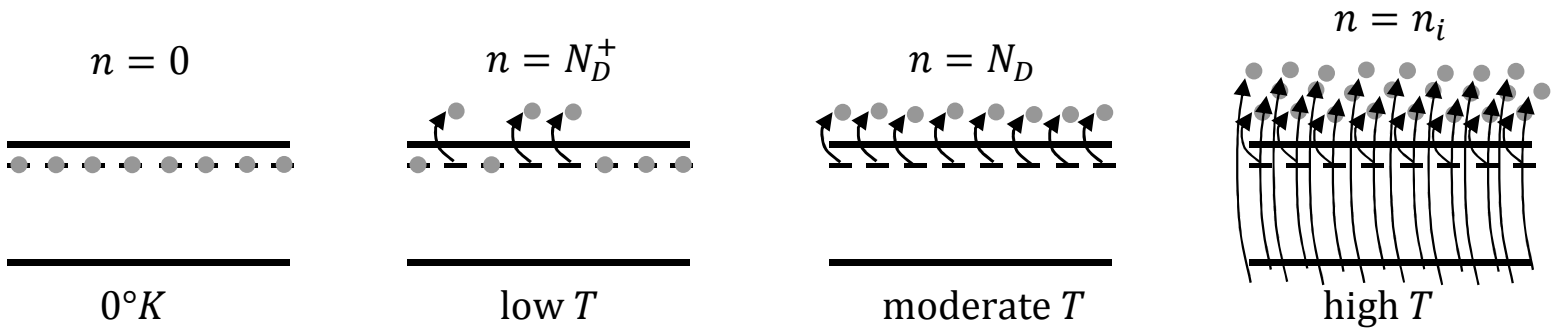
Carrier Concentration vs. Temperature

At room temperature, all the shallow dopants are ionized. (Extrinsic region)

When the temperature is decreased sufficiently (~100 K), some of the dopants are not ionized. (Freeze out region)

When the temperature is increased so high that the intrinsic carrier conc. approaches the active dopant conc. (T → Ti, > 450K for Si), the semiconductor is said to enter the intrinsic region.

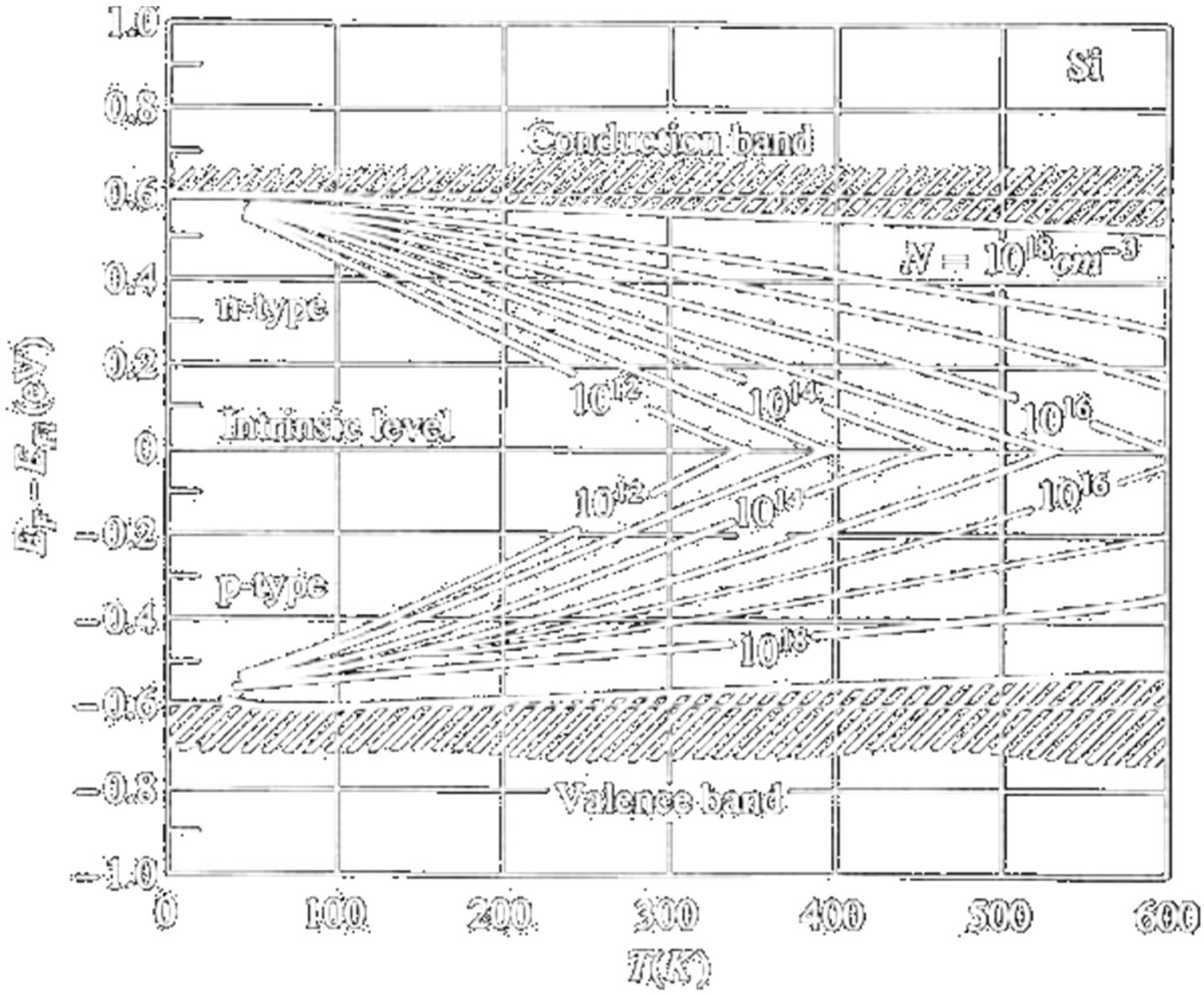




Fermi Level vs. Temperature

When the temperature is decreased, the Fermi level rises towards the donor level (N-type) and eventually gets above it.

When the temperature is increased, the Fermi level moves towards the intrinsic level.



Carrier Concentrations

Q: What is the hole concentration in an n-type semiconductor with $10^{15}\ cm^{-3}$ of donors? how much hole concentration will change if T increase by 60°C

Sol:

$$n = N_D = 10^{15} cm^{-3}$$

After increasing T by 60°C, n remains the same at $10^{15}\ cm^{-3}$ while p increases by about a factor of 2300 because

$$n_i^2 \propto e^{-E_G/kT}$$

Q: What is n if $p = 10^5 cm^{-3}$ in a p-type silicon wafer?

Sol:

$$n = \frac{n_i^2}{p} = \frac{10^{20}}{10^{17}} = 10^3 cm^{-3}$$

Dopant Ionization

Consider a phosphorus-doped Si sample at 300K with $N_D = 10^{17} cm^{-3}$. What fraction of the donors are not ionized?

Sol: Suppose all of the donor atoms are ionized.

then $$E_F = E_C - kT \ln \frac{N_c}{n} = E_C - 150meV$$

$$\frac{N_D^+}{N_D} = \frac{1}{1 + g_D e^{(E_F - E_D)/kT}} \qquad \frac{N_A^-}{N_A} = \frac{1}{1 + g_A e^{(E_A - E_F)/kT}}$$

$$g_D = 2 \qquad g_A = 4$$

Probability of non-ionization $$\sim \frac{1}{1 + {g_D}^{-1} e^{(E_F - E_D)/kT}}$$

$$= \frac{1}{1 + \frac{1}{2} e^{(150-45)/26}} = 0.034$$

Most General Eq.

Charge neutrality $n + N_A^- = p + N_D^+$

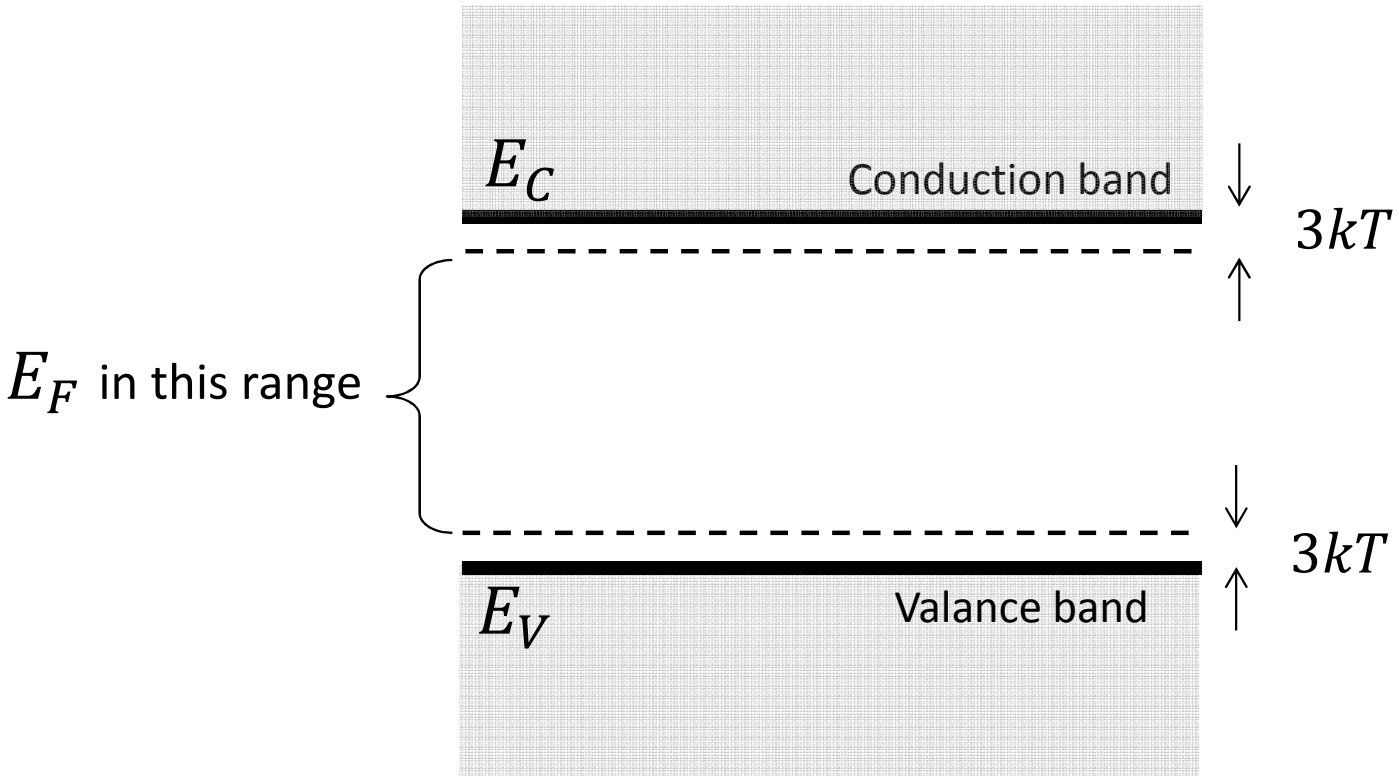
Mass law $np = n_i^2$

$$\frac{N_D^+}{N_D} = \frac{1}{1 + g_D e^{(E_F - E_D)/kT}} \qquad \frac{N_A^-}{N_A} = \frac{1}{1 + g_A e^{(E_A - E_F)/kT}}$$

Nondegenerately Doped Semiconductor

Recall that the expressions for n and p were derived using the Boltzmann approximation, i.e. we assumed

$$E_V + 3kT \leq E_F \leq E_C - 3kT$$



The semiconductor is said to be ***nondegenerately doped*** in this case.

Degenerately Doped Semiconductor

If a semiconductor is very heavily doped, the Boltzmann approximation is not valid.

In Si at $T = 300K$: $E_C - E_F < 3kT$ if $N_D > 1.6 \times 10^{18}\ cm^{-3}$

$E_F - E_V < 3kT$ if $N_A > 9.1 \times 10^{17}\ cm^{-3}$

The semiconductor is said to be ***degenerately doped*** in this case.

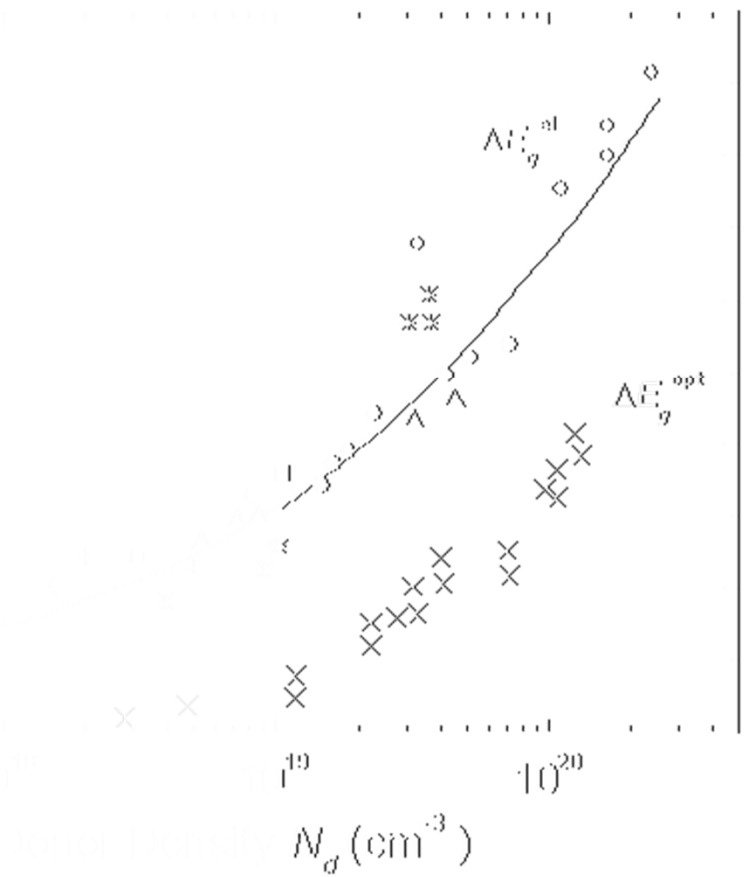
Terminology:

"n+" degenerately n-type doped. $E_F \sim E_C$

"p+" degenerately p-type doped. $E_F \sim E_V$

Band Gap Narrowing

If the dopant concentration is a significant fraction of the silicon atomic density, the energy-band structure is perturbed → the band gap is reduced by ΔE_G :



$$\Delta E_G \sim 3.5 \times 10^{-8} N^{1/3} \sqrt{\frac{300}{T}}$$

$$N = 10^{18} cm^{-3} \rightarrow \Delta E_G \sim 35 meV$$

$$N = 10^{19} cm^{-3} \rightarrow \Delta E_G \sim 75 meV$$