ECE606: Solid State Devices
Lecture 10

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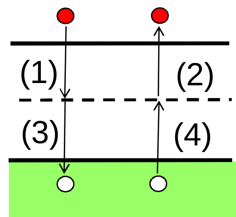
Outline

1) **Derivation of SRH formula**
2) Application of SRH formula for special cases
3) Direct and Auger recombination
4) Conclusion

Ref. ADF, Chapter 5, pp. 141-154

Sub-processes of SRH Recombination

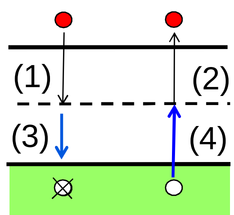


(1)+(3): one electron reduced from Conduction-band &
one-hole reduced from valence-band

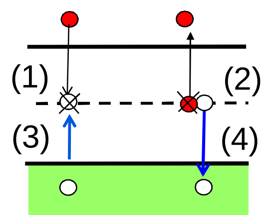
(2)+(4): one hole created in valence band and
one electron created in conduction band

SRH Recombination

Physical picture



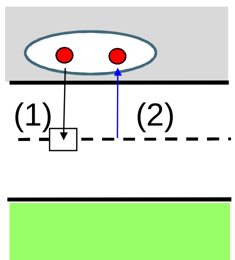
Equivalent picture

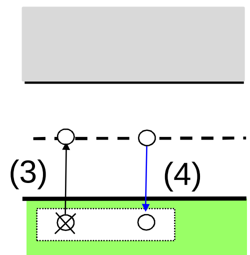


(1)+(3): one electron reduced from C-band &
one-hole reduced from valence-band

(2)+(4): one hole created in valence band &
one electron created in conduction band

Changes in electron and hole Densities

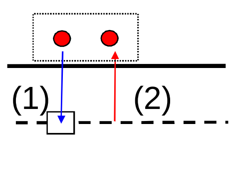




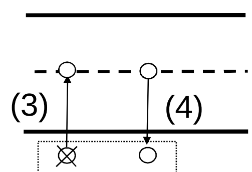
$$\left.\frac{\partial n}{\partial t}\right|_{1,2} = -c_n\, n\, p_T + e_n n_T \left(1 - f_c\right)$$

$$\left.\frac{\partial p}{\partial t}\right|_{3,4} = -c_p\, p\, n_T + e_p p_T f_\upsilon$$

Detailed Balance in *Equilibrium*



Subscripts 0 indicate equilibrium



$\left(1 - f_c\right) \approx 1$ Assume non-degenerate

$$\left.\frac{\partial n}{\partial t}\right|_{1,2} = -c_n\, n_0\, p_{T0} + e_n n_{T0}$$

$$0 = -c_n n_0 p_{T0} + e_n n_{T0}$$

$$e_n = c_n \frac{n_0 p_{T0}}{n_{T0}} \equiv c_n n_1$$

$$0 = -c_n \left(n_0 p_{T0} - n_{T0} n_1\right)$$

Fundamental concept of detailed balance, connecting 2 counteracting processes.

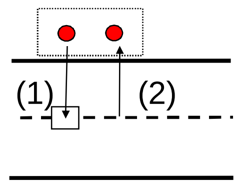
$$\left.\frac{\partial p}{\partial t}\right|_{3,4} = -c_p\, p\, n_T + e_p p_T$$

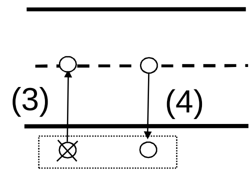
$$0 = -c_p p_0 n_{T0} + p_{T0} e_p$$

$$e_p \equiv \frac{c_p p_0 n_{T0}}{p_{T0}} = c_p p_1$$

$$0 = -c_p \left(p_0 n_{T0} - p_{T0} p_1\right)$$

Expressions for (n_1) and (p_1)



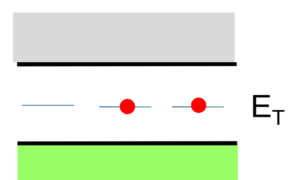


$$n_1 = \frac{n_0 p_{T0}}{n_{T0}} \qquad p_1 = \frac{p_0 n_{T0}}{p_{T0}}$$

$$n_1 p_1 = \frac{n_0 p_{T0}}{n_{T0}} \times \frac{p_0 n_{T0}}{p_{T0}} = n_0 p_0 = n_i^2$$

Expressions for (n_1) and (p_1)

Trap is like a donor! f00=empty trap prob.



$$n_{T0} = N_T\left(1 - f_{00}\right) = \frac{N_T}{1 + g_D e^{\beta(E_T - E_F)}}$$

$$\left(1 - f_{00}\right) = \frac{1}{1 + g\,exp} \qquad f_{00} = 1 - \frac{1}{1 + g\,exp}$$

$$f_{00} = \frac{g\,exp}{1 + g\,exp} \qquad f_{00} \Big/ 1 - f_{00} = \frac{g\,exp}{1 + g\,exp} \Big/ \frac{1}{1 + g\,exp} = g\,exp$$

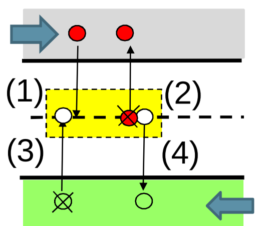
$$n_1 = \frac{n_0 p_{T0}}{n_{T0}} = n_0 \frac{\left(N_T f_{00}\right)}{N_T\left(1 - f_{00}\right)}$$

$$n_1 = n_i e^{\beta(E_F - E_i)} g_D e^{\beta(E_T - E_F)} = n_i g_D e^{\beta(E_T - E_i)}$$

$$p_1 n_1 = n_i^2$$

$$p_1 = n_i^2 / n_1 = n_i g_D^{-1} e^{\beta(E_i - E_T)}$$

Dynamics of Trap Population

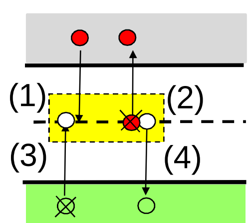


$$\frac{\partial n_T}{\partial t} = -\left.\frac{\partial n}{\partial t}\right|_{1,2} + \left.\frac{\partial p}{\partial t}\right|_{3,4}$$

$$= c_n\, n\, p_T \; - e_n n_T \; - c_p\, p\, n_T \; + e_p\, p_T$$

$$= c_n\left(n p_T - n_T n_1\right) - c_p\left(p\, n_T - p_T\, p_1\right)$$

Steady-state Trap Population



$$N_T = p_T + n_T$$

$$p_T = N_T - n_T$$

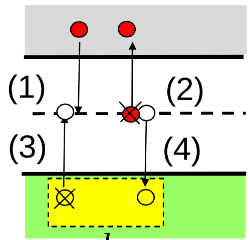
$$\frac{\partial n_T}{\partial t} = 0 = c_n\left(n p_T - n_T n_1\right) - c_p\left(p\, n_T - p_T\, p_1\right)$$

$$0 = c_n\left(n\left(N_T - n_T\right) - n_T n_1\right) - c_p\left(p n_T - \left(N_T - n_T\right) p_1\right)$$

$$n_T\left(c_n n + c_n n_1 + c_p p + c_p p_1\right) = c_n n N_T + c_p p_1 N_T$$

$$n_T = \frac{c_n N_T n + c_p N_T p_1}{c_n\left(n + n_1\right) + c_p\left(p + p_1\right)}$$

Net Rate of Recombination-Generation



$$p_T = N_T - n_T$$

$$n_T = \frac{c_n N_T n + c_p N_T p_1}{c_n (n+n_1) + c_p (p+p_1)} = \frac{c_n N_T n + c_p N_T p_1}{A}$$

$$p_1 n_1 = n_i^2$$

$$R = -\frac{dp}{dt} = c_p (p\, n_T - p_T p_1) = c_p (p\, n_T - N_T p_1 + n_T p_1)$$

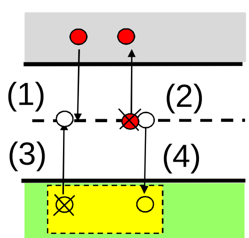
$$= c_p n_T (p + p_1) - c_p N_T p_1 = c_p (p+p_1) \frac{c_n N_T n + c_p N_T p_1}{A} - c_p N_T p_1 \frac{A}{A}$$

$$= \frac{c_p N_T}{A} (p c_n n + p_1 c_n n + p c_p p_1 + p_1 c_p p_1 - p_1 c_n n - p_1 c_n n_1 - p_1 c_p p - p_1 c_p p_1)$$

$$= \frac{c_p N_T c_n}{A} (pn - p_1 n_1) \qquad = \frac{np - n_i^2}{\left(\frac{1}{c_p N_T}\right)(n+n_1) + \left(\frac{1}{c_n N_T}\right)(p+p_1)}$$

τ_n → $\frac{1}{c_p N_T}$, τ_p → $\frac{1}{c_n N_T}$

Net Rate of Recombination-Generation



$$R = -\frac{dp}{dt} = c_p (p\, n_T - p_T p_1)$$

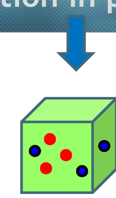
$$= \frac{np - n_i^2}{\left(\frac{1}{c_p N_T}\right)(n+n_1) + \left(\frac{1}{c_n N_T}\right)(p+p_1)}$$

τ_n → $\frac{1}{c_p N_T}$, τ_p → $\frac{1}{c_n N_T}$

Outline

1) Derivation of SRH formula
2) **Application of SRH formula for special cases**
3) Direct and Auger recombination
4) Conclusion

Case 1: Low-level Injection in p-type

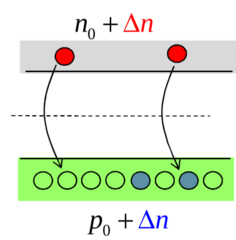


$$R = \frac{np - n_i^2}{\tau_p (n + n_1) + \tau_n (p + p_1)}$$

$$= \frac{(n_0 + \Delta n)(p_0 + \Delta n) - n_i^2}{\tau_p (n_0 + \Delta n + n_1) + \tau_n (p_0 + \Delta p + p_1)}$$

$$= \frac{\Delta n (n_0 + p_0) + \Delta n^2}{\tau_p (n_0 + \Delta n + n_1) + \tau_n (p_0 + \Delta p + p_1)}$$

$$= \frac{\Delta n (p_0)}{\tau_n (p_0)} = \frac{\Delta n}{\tau_n}$$



$$\Delta n^2 \approx 0$$

$$p_0 \gg \Delta n \gg n_0$$

Lots of holes, few electrons => independent of holes

Case 2: High-level Injection

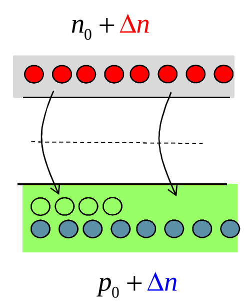
$$R = \frac{np - n_i^2}{\tau_p(n + n_1) + \tau_n(p + p_1)}$$

e.g. organic solar cells

$$= \frac{(n_0 + \Delta n)(p_0 + \Delta p) - n_i^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta p + p_1)}$$

$$= \frac{\Delta n(n_0 + p_0) + \Delta n^2}{\tau_p(n_0 + \Delta n + n_1) + \tau_n(p_0 + \Delta n + p_1)}$$

$$= \frac{\Delta n^2}{(\tau_n + \tau_p)\Delta n} = \frac{\Delta n}{(\tau_n + \tau_p)}$$



$$\Delta n \gg p_0 \gg n_0$$

Lots of holes, lots of electrons => dependent on both relaxations

High/Low Level Injection …

$$R_{high} = \frac{\Delta n}{(\tau_n + \tau_p)} \qquad \Delta n \gg p_0 \gg n_0$$

$$R_{low} = \frac{\Delta n}{\tau_p} \qquad p_0 \gg \Delta n \gg n_0$$

which one is larger and why?

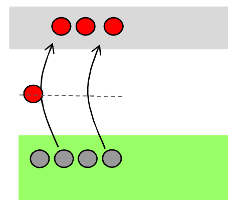
Case 3: Generation in Depletion Region

Depletion region – in PN diode: n=p=0

$$n \ll n_1 \quad p \ll p_1$$

$$R = \frac{np - n_i^2}{\tau_p\left(n + n_1\right) + \tau_n\left(p + p_1\right)}$$

$$= \frac{-n_i^2}{\tau_p\left(n_1\right) + \tau_n\left(p_1\right)}$$



NEGATIVE Recombination => Generation

n=p=0 << n_i => generation to create n,p

Equilibrium restoration!

Outline

1) Derivation of SRH formula
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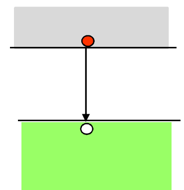
Band-to-band Recombination

$$R = B\left(np - n_i^2\right)$$ B is a material property



Direct recombination at low-level injection

$$n_0 \ll (\Delta n = \Delta p) \ll p_0$$

$$R = B\left[\left(n_0 + \Delta n\right)\left(p_0 + \Delta p\right) - n_i^2\right] \approx Bp_0 \times \Delta n$$

Direct generation in depletion region

$$n, p \sim 0$$

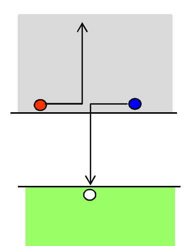
$$R = B\left(np - n_i^2\right) \approx -Bn_i^2$$

Auger Recombination

2 electron & 1 hole

$$R = c_n\left(n^2 p - n_i^2 n\right) + c_p\left(np^2 - n_i^2 p\right)$$

$$c_n, c_p \sim 10^{-29}\ \text{cm}^6/\text{sec}$$

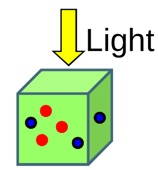


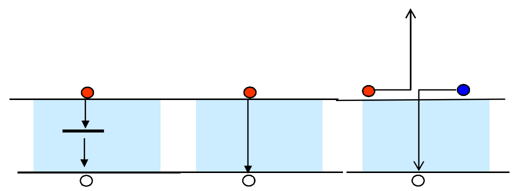
Auger recombination at low-level injection

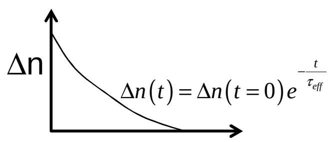
$$n_0 \ll (\Delta n = \Delta p) \ll \left(p_0 = N_A\right)$$

$$R \approx c_p N_A^2 \Delta n = \frac{\Delta n}{\tau_{auger}} \qquad \tau_{auger} = \frac{1}{c_p N_A^2}$$

Effective Carrier Lifetime







$$\Delta n(t) = \Delta n(t=0)\, e^{-\frac{t}{\tau_{eff}}}$$

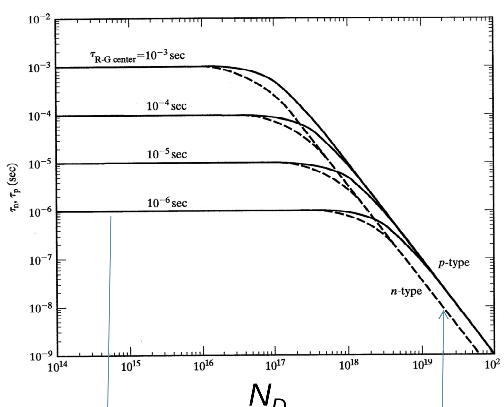
$$R = R_{SRH} + R_{direct} + R_{Auger}$$

$$= \Delta n\left(\frac{1}{\tau_{SRH}} + \frac{1}{\tau_{direct}} + \frac{1}{\tau_{Auger}}\right)$$

$$= \Delta n\left(c_n N_T + B N_D + c_{n,auger} N_D^2\right)$$

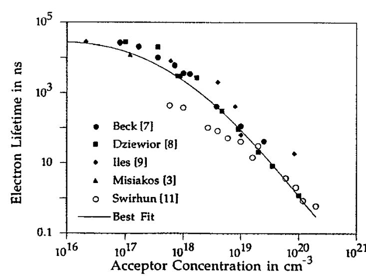
$$\tau_{eff} = \left(c_n N_T + B N_D + c_{n,auger} N_D^2\right)^{-1}$$

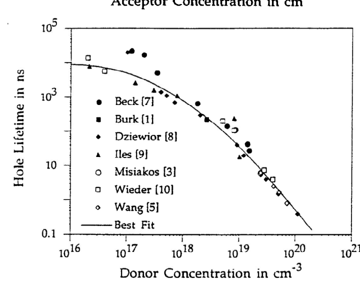
Effective Carrier Lifetime with all Processes



N_D

$$\tau_{eff} \approx c_{n,auger} N_D^{-2}$$





$$\tau_{eff} = \left(c_n N_T + B N_D + c_{n,auger} N_D^2\right)^{-1}$$

Elec. Dev. Lett., 12(8), 1991.

Conclusion

SRH is an important recombination mechanism in important semiconductors like Si and Ge.

SRH formula is complicated, therefore simplification for special cases are often desired.

Direct band-to-band and Auger recombination can also be described with simple phenomenological formula.

These expressions for recombination events have been widely validated by measurements.

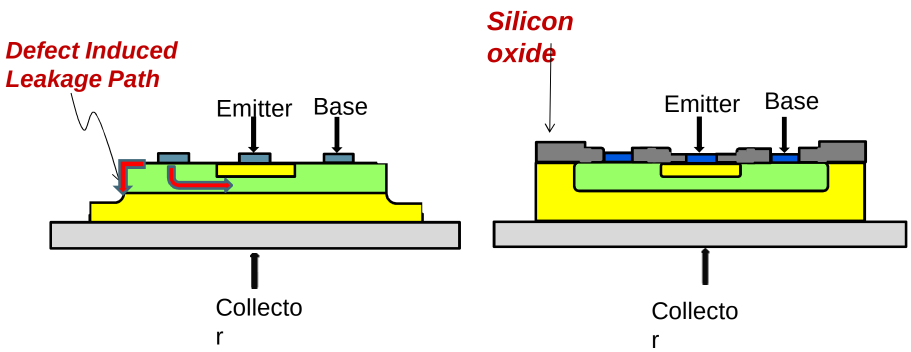
Outline

1) **Nature of interface states**
2) SRH formula adapted to interface states
3) Surface recombination in depletion region
4) Conclusion

REF: ADF, Chapter 5, pp. 154-167

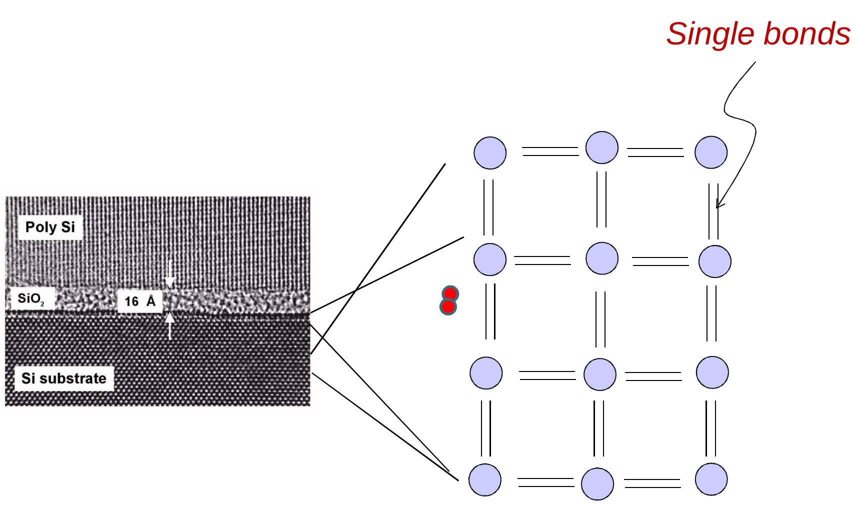
Surface states: Little bit of history

Hoerni's diagram of Mesa and planar transistors

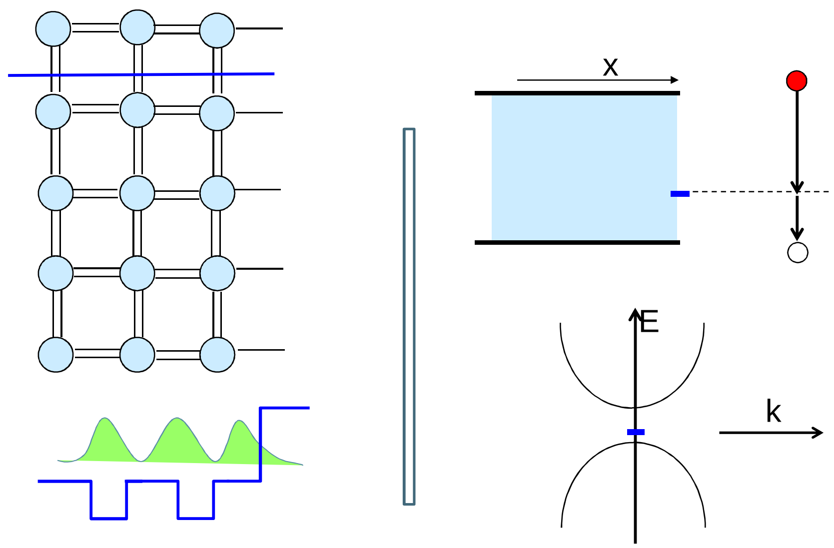


One of the fundamental advances in semiconductor history

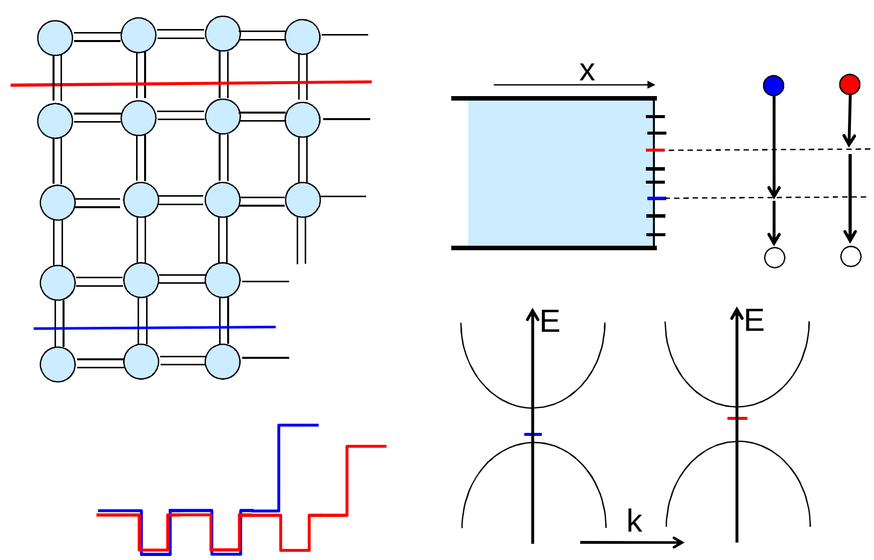
Atomic configuration of Surface States



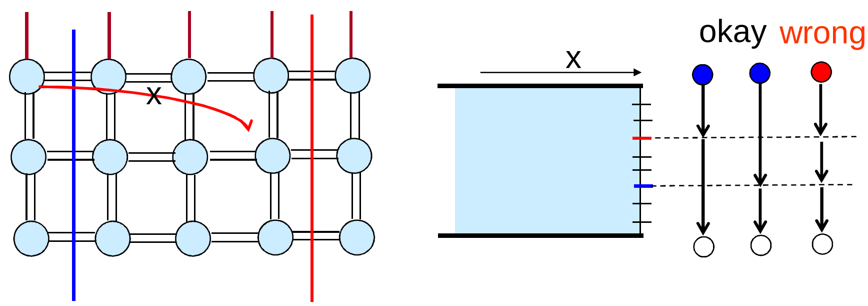
Surface States



Multiple Levels of Surface States



Multiple Levels of Surface States



Outline

1) Nature of interface states
2) **SRH formula adapted to interface states**
3) Surface recombination in depletion region
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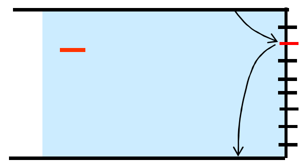
Surface Recombination Current

For single level bulk traps ….

$$R_{bulk} = \frac{np - n_i^2}{\frac{1}{c_p N_T}(n+n_1) + \frac{1}{c_n N_T}(p+p_1)} = \frac{(np-n_i^2)N_T}{\frac{1}{c_p}(n+n_1)+\frac{1}{c_n}(p+p_1)}$$

For single level interface trap at E …

$$R(E) = \frac{(n_s p_s - n_i^2) D_T(E)\,dE}{\frac{1}{c_{ps}}(n_s + n_{1s}) + \frac{1}{c_{ns}}(p_s + p_{1s})}$$



$$R = \int_{E_V}^{E_C} R(E)\,dE$$

Case 1: Minority Carrier Recombination

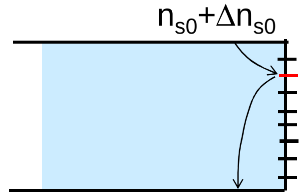
Donor doped

$$R(E) = \frac{\left[(n_{s0}+\Delta n_{s0})(p_{s0}+\Delta p_{s0}) - n_i^2\right] D_{IT}(E)\,dE}{\frac{1}{c_{ps}}(n_{s0}+\Delta n_{s0}+n_{1s}) + \frac{1}{c_{ns}}(p_{s0}+\Delta p_{s0}+p_{1s})}$$

$$= \frac{n_{s0}\Delta p_{s0} D_{IT}(E)\,dE}{n_{s0}\left[\frac{1}{c_{ps}} + \frac{n_{1s}}{c_{ps} n_{s0}} + \frac{p_{1s}}{c_{ns} n_{s0}}\right]}$$

$$= \frac{c_{ps}\Delta p_{s0} D_{IT}(E)\,dE}{\left[1 + \frac{n_{1s}}{n_{s0}} + \frac{c_{ps}}{c_{ns}}\frac{p_{1s}}{n_{s0}}\right]}$$

$n_{s0}+\Delta n_{s0}$



$p_{s0}+\Delta p_{s0}$

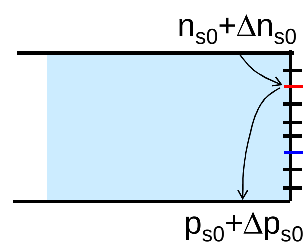
Consider the Denominator …

$$D = 1 + \frac{n_{1s}}{n_{s0}} + \frac{c_{ps}}{c_{ns}}\frac{p_{1s}}{n_{s0}} = 1 + \frac{n_{1s}}{N_D} + \frac{c_{ps}}{c_{ns}}\frac{p_{1s}}{N_D}$$

$$= 1 + \frac{n_i e^{(E-E_i)\beta}}{n_i e^{(E_F-E_i)\beta}} + \frac{c_{ps}}{c_{ns}}\frac{n_i e^{-(E-E_i)\beta}}{n_i e^{(E_F-E_i)\beta}}$$

$$= 1 + e^{(E-E_F)\beta} + \frac{c_{ps}}{c_{ns}} e^{(E_F-E\ )\beta}$$

$$= 1 + e^{x} + a e^{-x} \qquad x \equiv \beta\left(E - E_F\right)$$



Consider the Denominator …

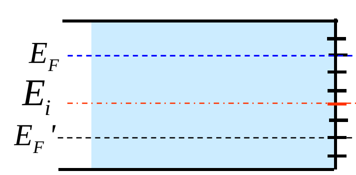
$$D = 1 + \frac{n_i e^{(E-E_i)\beta}}{N_D} + \frac{c_{ps}}{c_{ns}}\frac{n_i e^{-(E-E_i)\beta}}{N_D}$$

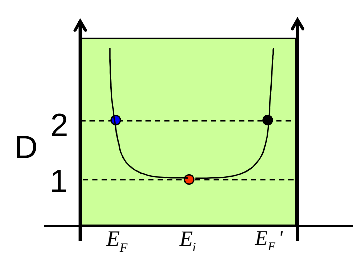
$$= 1 + e^{(E-E_F)\beta} + \frac{c_{ps}}{c_{ns}} e^{(E_F-E\ )\beta}$$

At $E = E_i \Rightarrow \quad D = 1 + \frac{n_i}{N_D} + \frac{c_{ps}}{c_{ns}}\frac{n_i}{N_D} \approx 1$

At $E = E_F > E_i, x = 0 \quad D = 1 + 1 + \frac{c_{ps}}{c_{ns}} \times small \approx 2$

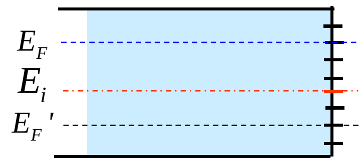
At $E = E_F' < E_i, \quad D = 1 + small + 1 = 2$



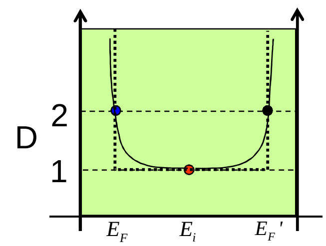


Approximate the Denominator ...

$$D = 1 + \frac{n_i e^{(E-E_i)\beta}}{N_D} + \frac{c_{ps}}{c_{ns}} \frac{n_i e^{-(E-E_i)\beta}}{N_D}$$



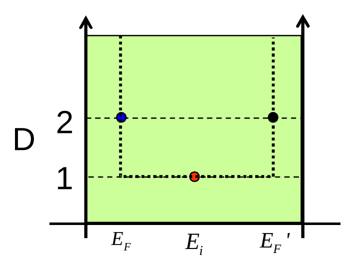
$$D \approx \begin{bmatrix} 1 & \text{for} \quad E_F \le E \le E_F' \\ \infty & \qquad\qquad \text{otherwise} \end{bmatrix}$$



Integrated Recombination

$$R = \int_{E_V}^{E_C} R(E) = \int_{E_V}^{E_C} \frac{c_{ps} \Delta p_{s0} D_{IT}(E)\, dE}{\left[1 + \frac{n_{1s}}{n_{s0}} + \frac{c_{ps}}{c_{ns}} \frac{p_{1s}}{n_{s0}}\right]}$$

$$\approx \int_{E_F'}^{E_F} c_{ps} \Delta p_{s0} D(E)\, dE$$

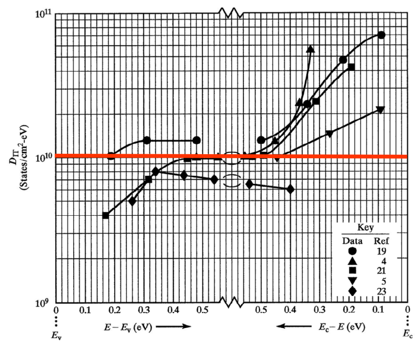


Surface Recombination Velocity

$$R \approx \int_{E_F}^{E_F} c_{ps} \Delta p_{s0} D_{IT}(E)\, dE$$

$$= c_{ps} D_{IT} \left(E_F - E_F^{'} \right) \Delta p_{s0}$$

$$= s_g \Delta p_{s0}$$



Surface recombination velocity

Outline

1) Nature of interface states
2) SRH formula adapted to interface states
3) **Surface recombination in depletion region**
4) Conclusion

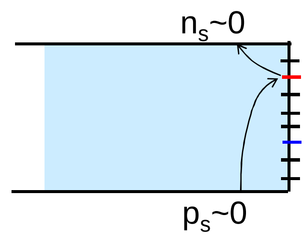
Case 2: Recombination in Depletion

$$R(E)=\frac{\left(n_s p_s - n_i^2\right)D_{IT}(E)\,dE}{\frac{1}{c_{ps}}\left(n_s+n_{1s}\right)+\frac{1}{c_{ns}}\left(p_s+p_{1s}\right)}$$

$$=-\frac{n_i}{\frac{n_i e^{(E-E_i)\beta}}{c_{ps}}+\frac{n_i e^{-(E-E_i)\beta}}{c_{ns}}}n_i D_{IT}(E)\,dE$$

$$=-c_{ns}D_{IT}n_i\frac{e^{(E-E_i)\beta}dE}{\frac{c_{ns}}{c_{ps}}e^{2(E-E_i)\beta}+1}$$

$$R=-c_{ns}D_{IT}n_i\int_{E_V}^{E_C}\frac{e^{(E-E_i)\beta}dE}{\frac{c_{ns}}{c_{ps}}e^{2(E-E_i)\beta}+1}$$



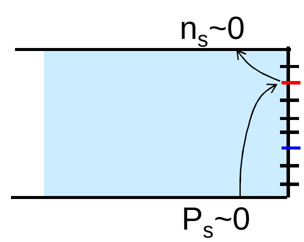
Case 2: Recombination in Depletion

$$R=-c_{ns}D_{IT}n_i\int_{E_V}^{E_C}\frac{e^{(E-E_i)\beta}dE}{\frac{c_{ns}}{c_{ps}}e^{2(E-E_i)\beta}+1}$$

$$=-c_{ns}D_{IT}n_i\int_{-\infty}^{+\infty}\frac{e^{(E-E_i)\beta}dE}{\frac{c_{ns}}{c_{ps}}e^{2(E-E_i)\beta}+1}$$

$$=-c_{ns}D_{IT}n_i\phi\sqrt{\frac{c_{ps}}{c_{ns}}}\int_{0}^{+\infty}\frac{dx}{x^2+1}$$

$$=-\sqrt{c_{ns}c_{ps}}\,D_{IT}n_i\beta\frac{\pi}{2}$$



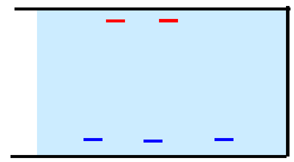
Why do donors/acceptors *not* act as R-G Centers?

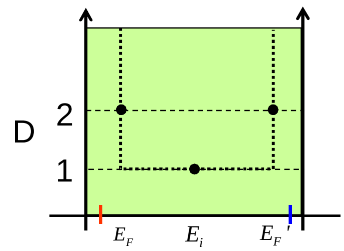
$$R\left(E_D\right)=\frac{c_{ps}\Delta p_{s0}D\left(E\right)dE}{\left[1+\frac{n_{1s}}{n_{s0}}+\frac{c_{ps}}{c_{ns}}\frac{p_{1s}}{n_{s0}}\right]}$$

$$=\frac{c_{ps}\Delta p_{s0}N_D}{D\left(E_D\right)}\rightarrow 0$$

$$R\left(E_A\right)=\frac{c_{ps}\Delta p_{s0}D\left(E\right)dE}{\left[1+\frac{n_{1s}}{n_{s0}}+\frac{c_{ps}}{c_{ns}}\frac{p_{1s}}{n_{s0}}\right]}$$

$$=\frac{c_{ps}\Delta p_{s0}N_A}{D\left(E_A\right)}\rightarrow 0$$





Summary

$$R=-\sqrt{c_{ns}c_{ps}}D_{IT}\beta\frac{\pi}{2}\times n_i$$ Interface (depletion)

$$R=c_{ps}D_{IT}\left(E_F-E_F'\right)\Delta p_s$$ Interface (minority)

$$R=c_pN_T\Delta p$$ Bulk (minority)