Session 2: Fundamentals

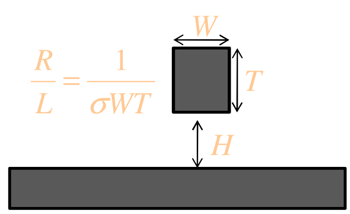
Introduction to VLSI Interconnect Design

Resistance

by definition is the ratio of potential difference of the wire ends to the total current flowing through it.

$$R \triangleq \frac{V_{12}}{I} = \frac{-\int_L \mathbf{E}.d\mathbf{l}}{\int_A \sigma \mathbf{E}.ds}$$

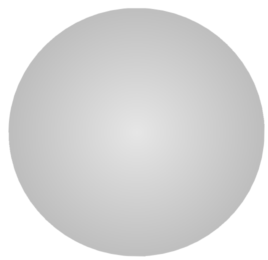
$$\frac{R}{L} = \frac{1}{\sigma W T}$$



Skin Effect

At high frequencies, current tends to distribute near the surface of a conductor

$$\delta \triangleq \frac{1}{\sqrt{\pi f \mu \sigma}}$$



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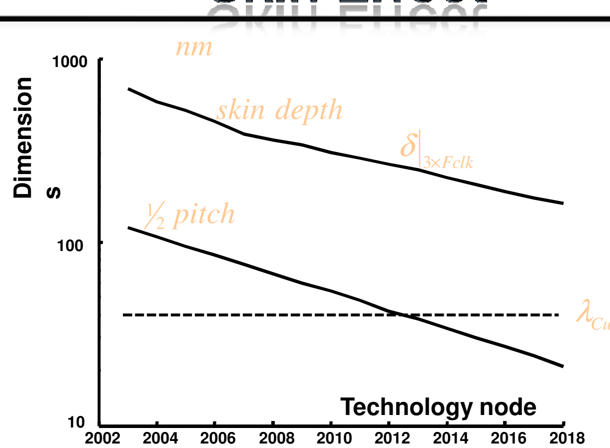
Skin Effect



Scale of GSI interconnections is continually shrinking toward dimensions comparable with the mean free path of the electrons.

At the same time, interconnects operate at higher frequencies such that skin depth becomes in the same order of mfp of electrons.

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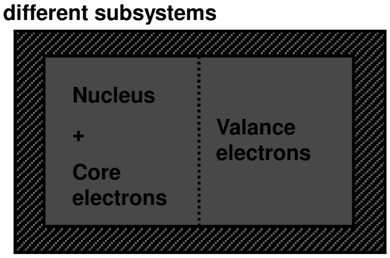
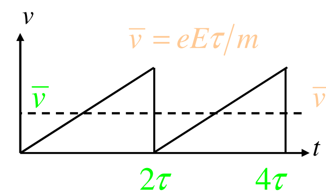
Electrons in Metals

In D-L-S model the metal is divided into 2 different subsystems



The kinetic theory of gas is applied to the metal gas



$$J = ne\overline{v} = \frac{ne^2\tau}{m^*}E = \sigma E$$

$$\lambda = v_F\tau$$

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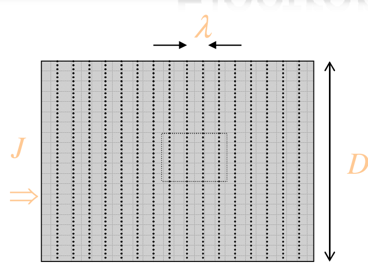
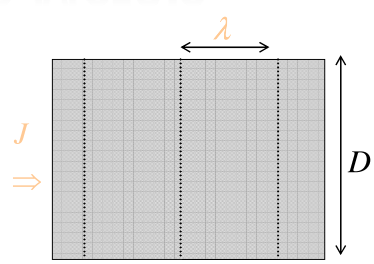
Electrons in Metals



$$\lambda << D$$

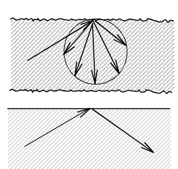
$$\rho \approx \rho_b$$



$$\lambda \approx D$$

$$\rho \approx \rho_b(1 + (1-p)\frac{\lambda}{D})$$

p = ***specularity parameter*** (the fraction of electrons that have elastic collisions at the wire surfaces) (*0<p<1*)



diffuse scattering $p = 0$

specular scattering $p = 1$

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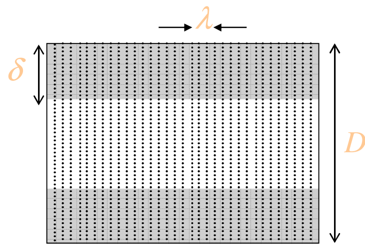
Anomalous Skin Effect

Skin effect

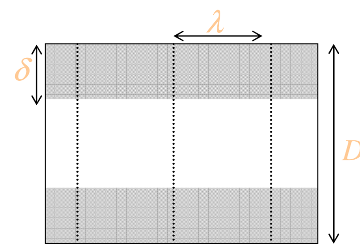


$D > \delta >> \lambda$

$\delta = \sqrt{\rho_b / \pi \mu f}$

$R \propto f^{1/2}$

Anomalous skin effect



$D > \delta \approx \lambda$

$\delta = \sqrt{\rho / \pi \mu f} \rightsquigarrow \rho = \rho_b (1 + \frac{\lambda}{\delta})$

$R \propto f^{2/3}$

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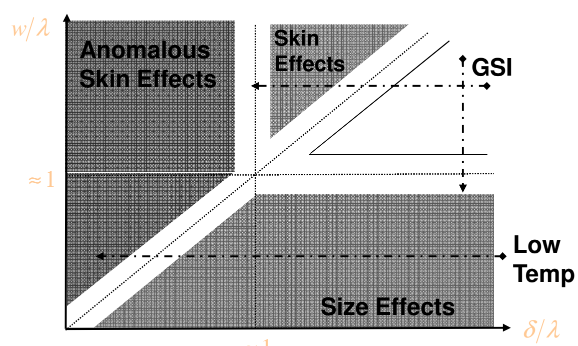
Validity of Asymptotic Solutions



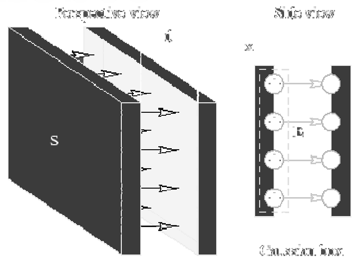
@ room temperature:

$\lambda_{Cu} \approx 39nm \quad \lambda_{Al} \approx 21nm$

$\delta_{Cu}|_{10GHz} \approx 600nm$

Capacitance

- A capacitor is a passive electronic component that stores energy in the form of an electrostatic field.



$$C \triangleq \frac{Q}{V_{12}} = \frac{\oint_S \mathbf{D} \cdot ds}{-\int_L \mathbf{E} \bullet d\mathbf{l}}$$

In its simplest form, a capacitor consists of two conducting plates separated by an insulating material called the dielectric. The capacitance is directly proportional to the surface areas of the plates, and is inversely proportional to the separation between the plates.

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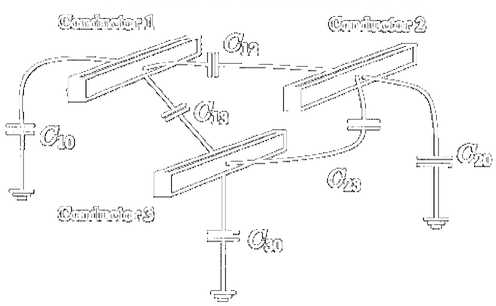
Capacitance



$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{pmatrix} C_{10}+C_{12}+C_{13} & -C_{12} & -C_{13} \\ -C_{12} & C_{20}+C_{12}+C_{23} & -C_{23} \\ -C_{13} & -C_{23} & C_{30}+C_{13}+C_{23} \end{pmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

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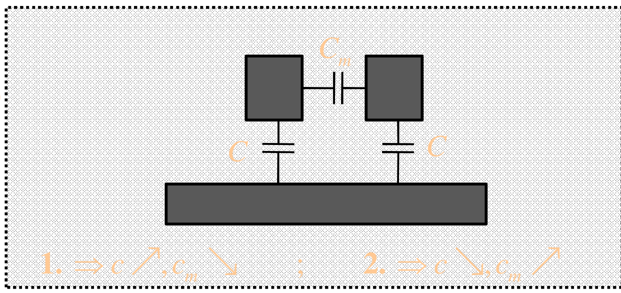
Capacitance

Volume-based method: (finite-element, finite-difference)

B.C.

1. $v = 0$

2. $dv/dn = 0$



+ : accuracy, any complex structure

- : time consuming

Software: Maxwell, HFSS, Raphael

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Capacitance

Surface-based method: (integral equation)

Green's function $G(r, r') = \dfrac{1}{4\pi \|r - r'\|}$

Integral equation
(panel method, method of moments)

$$V = \int_{surface} G(r, r')\sigma(r')da'$$

$$Q = \int_{surface} \sigma(r')da'$$

Software: Boundary Elements Method, FASTHENRY

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Capacitance

Random-walk method: (stochastic)

$$V_{center} = \frac{1}{N}\sum_{i=1}^{N} V_{square}(x_i)$$

Software: Random Logic Corp, QuicCap

Random walk: best for self cap for complicated net
Surface based: best for small coupling capacitance
Volume based: best for dealing with multiple dielectrics

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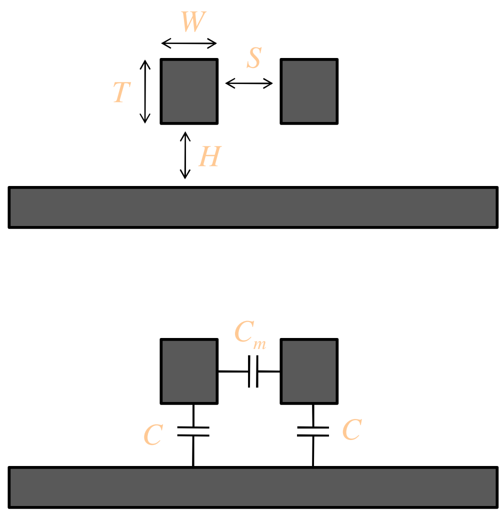
Capacitance



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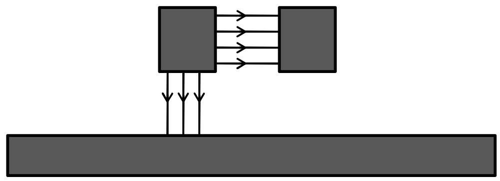
Parallel Plate Approximation



$$C = \varepsilon \frac{W}{H} \qquad C_m = \varepsilon \frac{T}{S}$$

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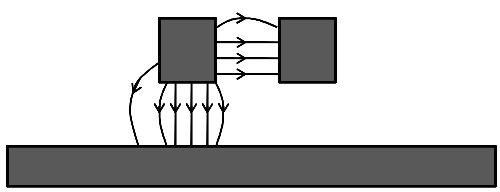
Sakurai Formula



$$\frac{C}{\varepsilon} = 1.15\frac{W}{H} + 2.8\left(\frac{T}{H}\right)^{0.222}$$

$$\frac{C_m}{\varepsilon} = \left[1.82\left(\frac{T}{H}\right)^{1.08} + 2.8\left(\frac{W}{T}\right)^{0.32}\right]\left(\frac{S}{H} + 0.43\right)^{-1.38}$$

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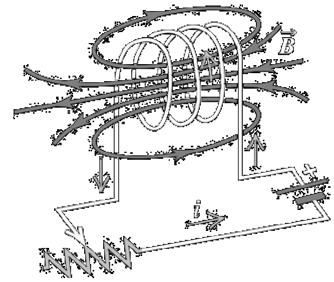
Inductance

- An inductor is a passive electronic component that stores energy in the form of a magnetic field.



In its simplest form, an inductor consists of a wire loop or coil. The inductance is directly proportional to the number of turns in the coil. Inductance also depends on the radius of the coil and on the type of material around which the coil is wound

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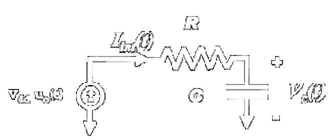
Lumped RC model



$$v_c(t) = V_{dd}(1 - e^{-t/RC})$$

$$t_{0.5} = 0.693RC$$

$$t_{0.9} = 2.3RC$$

Using parallel plate approximation

$$t_{0.5} = 0.693\frac{\rho\varepsilon L^2}{TH}$$

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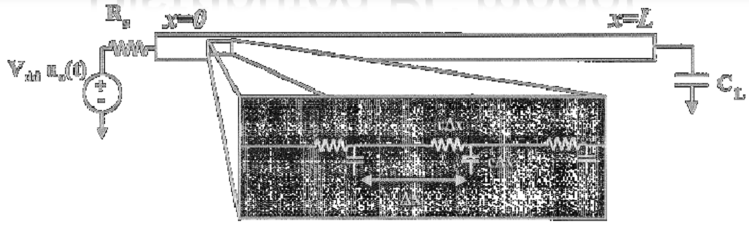
Distributed RC model



Differential Equations

$$\frac{\partial^2}{\partial x^2}V(x,t) = rc\frac{\partial}{\partial t}V(x,t)$$

Boundary Conditions

$$\frac{V_{in}}{s} - I(x=0,s)R_s = V(x=0,s)$$

$$I(x=L,s) = C_L s V(x=L,s)$$

$$V(L,s) = \frac{V_{in}(s)}{\sqrt{src}\left(\frac{C_L}{c}+\frac{R_s}{r}\right)\sinh\sqrt{sRC} + \left(1+R_sC_Ls\right)\cosh\sqrt{sRC}}$$

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Distributed RC model

$$V(t,L) = V_{dd}\left(1+K_1e^{\delta_1 t}+K_2e^{\delta_2 t}+\cdots\right)$$

$$K_1 = -1.01\frac{R_T+C_T+1}{R_T+C_T+\frac{\pi}{4}}$$

$$\sigma_1 = \frac{-\delta_1}{RC} = \frac{1.04}{R_TC_T+R_T+C_T+(2/\pi)^2}$$

$$t_\upsilon = RC(R_TC_T+R_T+C_T+(2/\pi)^2)\ln(\frac{1}{1-\upsilon})+0.1RC$$

$$t_{0.9} = 2.3(R_sC_L+R_sC+RC_L)+RC$$

$$t_{0.5} = 0.69(R_sC_L+R_sC+RC_L)+0.38RC$$

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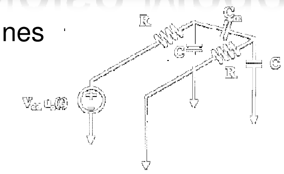
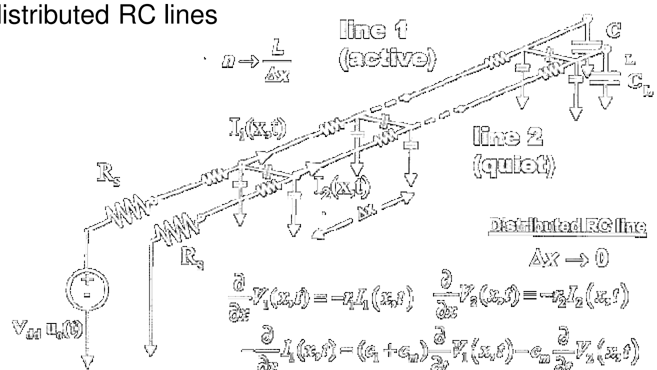
Noise Model

Two coupled lumped RC lines



Two coupled distributed RC lines



$$\frac{\partial}{\partial x}V_1(x,t) = -r_1 I_1(x,t) \quad \frac{\partial}{\partial x}V_2(x,t) = -r_2 I_2(x,t)$$

$$-\frac{\partial}{\partial x}I_1(x,t) = (c_1+c_m)\frac{\partial}{\partial t}V_1(x,t) - c_m\frac{\partial}{\partial t}V_2(x,t)$$

$$-\frac{\partial}{\partial x}I_2(x,t) = (c_1+c_m)\frac{\partial}{\partial t}V_2(x,t) - c_m\frac{\partial}{\partial t}V_1(x,t)$$

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Noise Model

Combining these equations

$$\frac{1}{r_1}\frac{\partial^2}{\partial x^2}V_1(x,t) = (c_1+c_m)\frac{\partial}{\partial t}V_1(x,t) - c_m\frac{\partial}{\partial t}V_2(x,t)$$

$$\frac{1}{r_2}\frac{\partial^2}{\partial x^2}V_2(x,t) = (c_2+c_m)\frac{\partial}{\partial t}V_2(x,t) - c_m\frac{\partial}{\partial t}V_1(x,t)$$

Assuming $r_1 = r_2 = r$ and $c_1 = c_2 = c$ simplifies to:

$$\frac{\partial^2}{\partial x^2}(V_1+V_2) = rc\frac{\partial}{\partial t}(V_1+V_2)$$

$$\frac{\partial^2}{\partial x^2}(V_1-V_2) = r(c+2c_m)\frac{\partial}{\partial t}(V_1-V_2)$$

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Noise Model

Boundary conditions

$$V_{dd}u(t) - I_1(0,t)R_s = V_1(0,t) \quad ; \quad -I_2(0,t)R_s = V_2(0,t)$$

$$I_1(L,s) = C_L \frac{\partial}{\partial t} V_1(L,s) \quad ; \quad I_2(L,s) = C_L \frac{\partial}{\partial t} V_2(L,s)$$

Transformation

$$V_- = (V_1 - V_2)/\sqrt{2} \quad ; \quad V_+ = (V_1 + V_2)/\sqrt{2}$$

Solution:

$$\frac{\partial^2}{\partial x^2} V_+ = rc \frac{\partial}{\partial t} V_+ \qquad \frac{\partial^2}{\partial x^2} V_- = r(c + 2c_m) \frac{\partial}{\partial t} V_-$$

$$\frac{V_{dd}}{\sqrt{2}} u(t) - I_+(0,t)R_s = V_+(0,t) \qquad \frac{V_{dd}}{\sqrt{2}} u(t) - I_-(0,t)R_s = V_-(0,t)$$

$$I_+(L,t) = C_L \frac{\partial}{\partial t} V_+(L,t) \qquad I_-(L,t) = C_L \frac{\partial}{\partial t} V_-(L,t)$$

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Noise Model

Sakurai single line solution

$$K_1 = -1.01 \frac{R_T + C_T + 1}{R_T + C_T + \frac{\pi}{4}}$$

$$\sigma_1 = \frac{1.04}{R_T C_T + R_T + C_T + (2/\pi)^2}$$

$$V(t, x = l) \approx V_{dd}(1 + K_1 \exp(\tfrac{-\sigma_1 t}{RC}))$$

Plus solution

$$V_+(t, x = L) \approx \frac{V_{dd}}{\sqrt{2}} \left(1 - 1.01 \frac{R_T + C_T^+ + 1}{R_T + C_T^+ + \frac{\pi}{4}} \exp\left(\frac{-1.04t}{RC} \frac{1}{R_T C_T^+ + R_T + C_T^+ + (2/\pi)^2}\right) \right)$$

Minus solution

$$V_-(t, x = L) \approx \frac{V_{dd}}{\sqrt{2}} \left(1 - 1.01 \frac{R_T + C_T^- + 1}{R_T + C_T^- + \frac{\pi}{4}} \exp\left(\frac{-1.04t}{R(C + 2C_m)} \frac{1}{R_T C_T^- + R_T + C_T^- + (2/\pi)^2}\right) \right)$$

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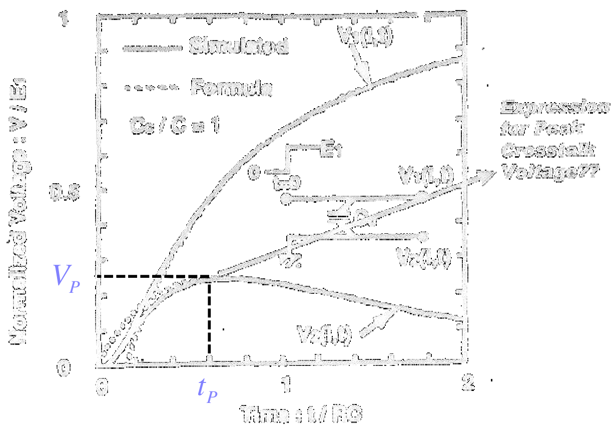
Noise Model

Active line $V_1 = \frac{V_+ + V_-}{\sqrt{2}}$ Quiet line $V_2 = \frac{V_+ - V_-}{\sqrt{2}}$



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Noise Model

Solving for t :

$$t_P = \ln\left[\frac{K_1^-}{K_1^+}\frac{\sigma_1^-}{\sigma_1^+}\frac{C}{C+2C_m}\right] \Big/ \left(\frac{\sigma_1^-}{R(C+2C_m)} - \frac{\sigma_1^+}{RC}\right)$$

Assuming $C_L << C$ simplifies to:

$$V_P = 1.01\frac{V_{dd}}{2}\frac{R_T+1}{R_T+\frac{\pi}{4}}\left(\frac{C}{C+2C_m}\right)^{C/2C_m}\left(\frac{2C_m}{C+2C_m}\right) \approx \frac{1}{2}\frac{C_m}{C+C_m}$$

$$\frac{V_P}{V_{dd}} \approx \frac{1}{2}\frac{C_m}{C+C_m}$$

parallel plate approximation

$$\frac{1}{2}\frac{T/S}{W/H + T/S}$$

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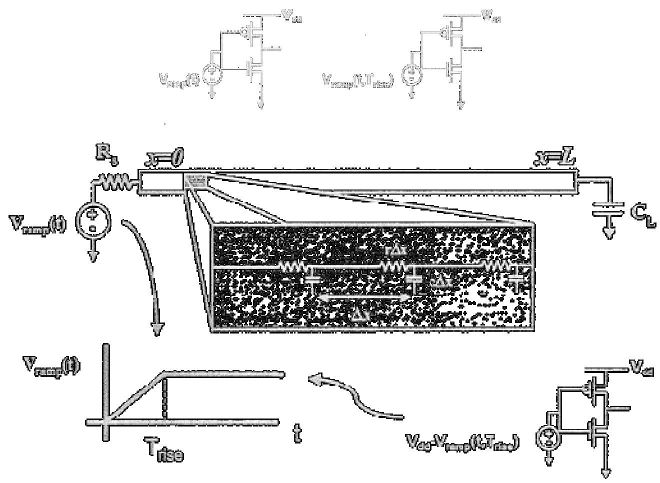
Ramp Input

Finite rise time?!



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Ramp Input

Solution:
Transient voltage

Region I: $t \le T_{rise}$

$$\frac{V(t)}{V_{dd}} = \left(t - B_0\left(1 - e^{\frac{-\sigma_0 t}{RC}}\right)\right)\frac{1}{T_{rise}}$$

Region II: $t > T_{rise}$

$$\frac{V(t)}{V_{dd}} = \left(1 - B_0 e^{\frac{-\sigma_0 t}{RC}} \frac{\left(e^{\frac{\sigma_0 T_{rise}}{RC}} - 1\right)}{T_{rise}}\right)$$

parameters

$$\sigma_0 = \frac{1.04}{\frac{R_S C_L}{rcL^2} + \frac{R_S}{rL} + \frac{C_L}{cL} + \left(\frac{2}{\pi}\right)^2}$$

$$B_0 = 1.01 \frac{rcL^2}{\sigma_0} \frac{\frac{R_S}{rL} + \frac{C_L}{cL} + 1}{\frac{R_S}{rL} + \frac{C_L}{cL} + \frac{\pi}{4}}$$

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Ramp Input

Time delay expressions:

Region I

$$t_v + e^{\frac{-\sigma_o t_v}{RC}} = T_{rise} v + B_o$$

$T_{rise} >> RC$

$$t_v \approx \frac{T_{rise} v + B_o - 1}{1 - \frac{\sigma_o}{RC}} \approx v T_{rise}$$

Region II

$RC >> T_{rise}$

$$t_v \approx \frac{RC}{\sigma_o} \ln\left[B_o\left(e^{\frac{\sigma T_{rise}}{RC}} - 1\right)\right] + \frac{RC}{\sigma_o} \ln\left[\frac{1}{(1-v)T_{rise}}\right]$$

As $T_{rise} \rightarrow 0$ converges to Sakurai

$$t_v = \frac{RC}{\sigma_0} \ln\left[\frac{B_o\left(e^{\frac{-\sigma_0 T_{rise}}{RC}} - 1\right)}{(1-v) T_{rise}}\right]$$

$$t_v = \left(R_s C_L + R_s C + R C_L + \left(\frac{2}{\pi}\right)^2 RC\right) \ln\left(\frac{1}{1-v}\right)$$

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Ramp Input

Generalized delay formula for $RC > T_{rise}$

$$t_{0.5} = \left(R_s C_L + R_s C + R C_L + \left(\frac{2}{\pi}\right) RC\right) \ln\left[\frac{B_o\left(e^{\frac{\sigma_0 T_{rise}}{RC}} - 1\right)}{0.5 T_{rise}}\right] + 0.1 RC$$

Coupled line solutions:

Region I: $t \leq T_{rise}$

$$\frac{V_2}{V_{dd}} = \frac{C_m (R + 2R_s) + B_1' e^{-\frac{\sigma_1 t}{RC}} - B_{-1}' e^{-\frac{\sigma_{-1} t}{RC}}}{2 T_{rise}}$$

Region II: $t > T_{rise}$

$$\frac{V_2}{V_{dd}} = \frac{-B_{-1}' e^{-\frac{\sigma_{-1} t}{RC}} \left(-e^{\frac{\sigma_{-1}}{RC} T_{rise}} + 1\right) + B_1' e^{-\frac{\sigma_1 t}{RC}} \left(1 - e^{\frac{\sigma_1}{RC} T_{rise}}\right)}{2 T_{rise}}$$

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Ramp Input

Hspice comparison

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Ramp Input

Peak crosstalk expression

$$\frac{V_{peak}}{V_{dd}} = \frac{\chi B_o^+}{T_{rise}}\left(1 - e^{-\frac{\sigma_o^+ T_{rise}}{B_o^+ rc^+}}\right)\left(1 - \frac{\sigma_o^+ c^-}{\sigma_o^- c^+}\right)\left(\frac{\sigma_o^+ c^- B_o^+\left(1 - e^{-\frac{T_{rise}\sigma_o^+}{B_o^+ rc^+}}\right)}{\sigma_o^- c^+ B_o^-\left(1 - e^{-\frac{T_{rise}\sigma_o^-}{B_o^- rc^-}}\right)}\right)^{\frac{\frac{\sigma_o^+}{c^+}}{\left(\frac{\sigma_o^+}{c^+} - \frac{\sigma_o^-}{c^-}\right)}}$$

$$\sigma_o^{\pm} = \frac{1.04}{\frac{R_T C_L}{rc^{\pm} L^2} + \frac{R_T}{rL} + \frac{C_L}{c^{\pm} L} + \left(\frac{2}{\pi}\right)^2}$$

$$B_o^{\pm} = 1.01\frac{rc^{\pm} L^2}{\sigma_o^{\pm}}\frac{\frac{R_T}{rL} + \frac{C_L}{c^{\pm} L} + 1}{\frac{R_T}{rL} + \frac{C_L}{c^{\pm} L} + \frac{\pi}{4}}$$

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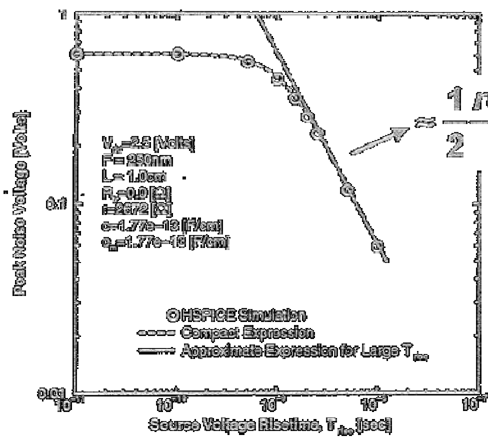
Ramp Input

Hspice comparison



$$\approx \frac{1}{2}\frac{rc_m L^2}{T_{rise}}$$

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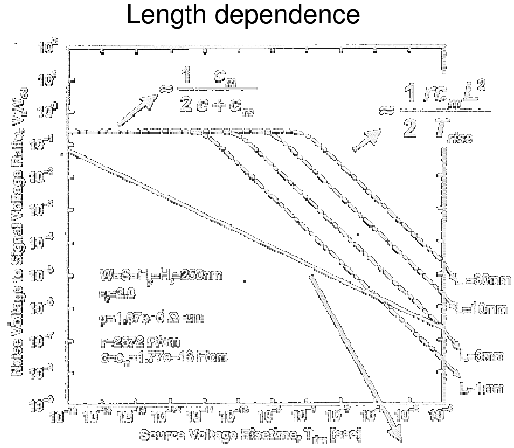
Ramp Input

Scaling independent:

$$\frac{V_p}{V_{dd}} \approx \frac{1}{2}\frac{C_m}{C+C_m}$$ parallel plate approximation $$\frac{1}{2}\frac{T/S}{W/H+T/S}$$

Length dependence



Fundamental Thermal Noise Limit

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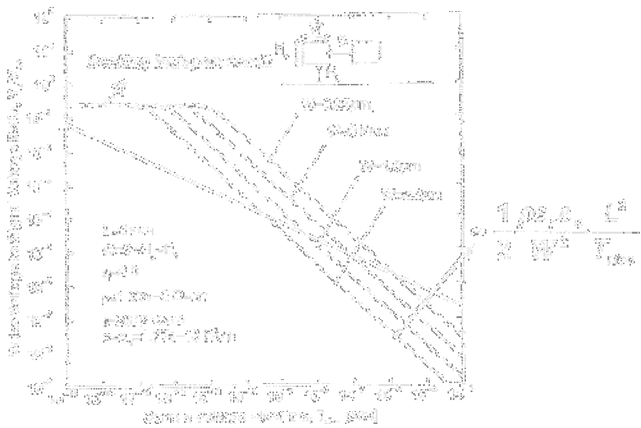
Ramp Input

Scaling dependence



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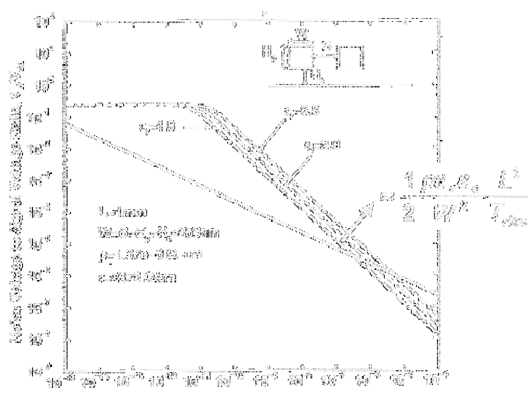
Ramp Input

Material dependence



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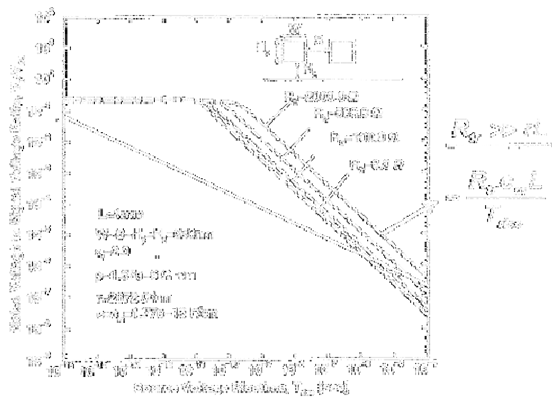
Ramp Input

Driver resistance dependence



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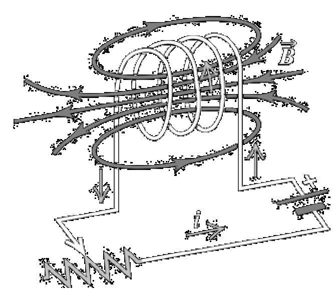
Inductance

- An inductor is a passive electronic component that stores energy in the form of a magnetic field.



In its simplest form, an inductor consists of a wire loop or coil. The inductance is directly proportional to the number of turns in the coil. Inductance also depends on the radius of the coil and on the type of material around which the coil is wound

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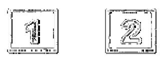
Inductance Formulas

Inductance of rectangular wires with return path at infinity



$$L_{11,22} = \frac{\mu_o L}{2\pi}\left[\ln\left(\frac{2L}{(W+H_c)}\right) + \frac{1}{2} + 0.2235\left(\frac{W+H_c}{L}\right)\right]$$

$$L_{12,21} = \frac{\mu_o L}{2\pi}\left[\ln\left(\frac{2L}{S}\right) - 1 + \frac{S}{L}\right]$$

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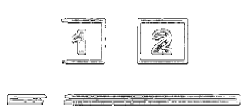
Inductance Formulas

Inductance of rectangular wires with return path in
perfect ground plane



$$L_{11,22} = 0.2\ln\left[\frac{2\left(H_s + \frac{H_c}{2}\right)\pi}{(W+H_c)}\right] \text{ nH/mm}$$

$$L_{12,21} = 0.2\ln\left[1 + \left(\frac{2\left(H_s + \frac{H_c}{2}\right)}{D}\right)^2\right] \text{ nH/mm}$$

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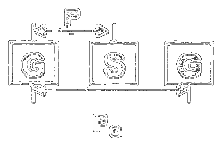
Inductance Formulas

Loop inductance for coplanar ground lines



$\alpha = P_G / P$

$$l_{coplanar} = \frac{\mu_0}{2\pi}\left[\ln\left(\frac{\pi P}{W_s + t}\right) + \frac{1}{2}\ln\left(\frac{\pi P}{W_g + H_g}\right) + \frac{1}{2}\ln\left(1 - \frac{1}{\alpha}\right) + \frac{1}{2}\frac{\ln\left(\frac{\alpha P_G}{(\alpha - 1)(W_g + H_g)}\right)}{\ln\left(\frac{\pi P_G}{W_g + H_g}\right)}\ln(\alpha - 1)\right]$$

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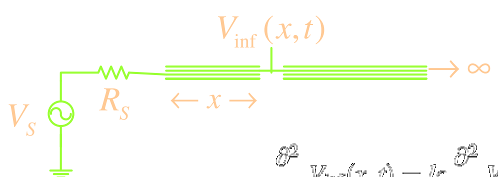
RLC model – Semi Infinite



$$\frac{\partial^2}{\partial x^2}V_{\inf}(x,t) = lc\frac{\partial^2}{\partial t^2}V_{\inf}(x,t) + rc\frac{\partial}{\partial t}V_{\inf}(x,t)$$

In its simplest

$$\frac{\partial^2}{\partial x^2}V_{\inf}(x,s) = lcs\left(s + \frac{r}{l}\right)V_{\inf}(x,s)$$

$$V_{\inf}(x,s) = A\ \exp\left\{-x\sqrt{lc}\sqrt{s\left(s + \frac{r}{l}\right)}\right\} + B\ \exp\left\{x\sqrt{lc}\sqrt{s\left(s + \frac{r}{l}\right)}\right\}$$

$$V_{\inf}(x,s) = V_S(s)\frac{Z(s)}{Z(s) + R_S}\exp\left\{-x\sqrt{lc}\sqrt{s\left(s + \frac{r}{l}\right)}\right\}$$

$$Z(s) = \sqrt{\frac{r + sl}{sc}} = Z_0\sqrt{\frac{s + r/l}{s}}$$

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RLC model – Semi Infinite

$$V_{\inf}(x,t) = V_S\left(\frac{Z_0}{Z_0+R_S}\right)e^{-\sigma t}[I_0(\sigma\sqrt{t^2-(x\sqrt{lc})^2})+$$

$$\frac{1}{1-\Gamma}\sum_{k=1}^{\infty} I_k(\sigma\sqrt{t^2-(x\sqrt{lc})^2})\left(\frac{t-x\sqrt{lc}}{t+x\sqrt{lc}}\right)^{k/2}\left[4-\Gamma^{k-1}(1+\Gamma)^2\right]] \quad u_0(t-x\sqrt{lc})$$

where $\sigma = r/2l \quad ; \quad \Gamma = \frac{R_S - Z_0}{R_S + Z_0}$

Note that: $V_{\inf}(x, x\sqrt{lc}) = V_S\left(\frac{Z_0}{Z_0+R_S}\right)e^{-rx/2Z_0}$

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RLC model – Finite

$$V_{fin}(\ell,s) = 2V_{\inf}(\ell,s) + 2\sum_{n=1}^{q}\left(\frac{R_S - Z(s)}{R_S + Z(s)}\right)^n V_{\inf}[(2n+1)\ell, s]$$

Delay model:

$$T_{d,RLC} = \begin{cases} \frac{\ell}{\sqrt{lc}} & \text{for } \frac{R}{Z_0} \le \ln\left[\frac{4Z_0}{R_S+Z_0}\right] \text{ and } R_S < 3Z_0 \\ 0.693R_S c\ell + 0.377rc\ell^2 & \text{for } \frac{R}{Z_0} \ge 2\ln\left[\frac{4Z_0}{R_S+Z_0}\right] \text{ or } R_S > 3Z_0 \end{cases}$$

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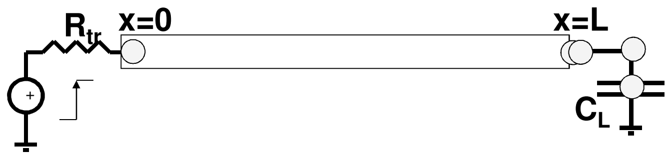
RLC model – Delay Model

Capacitive load

Delay model:

$$t_d \approx \max\left[t_F, 0.37rcL^2 + 0.69R_S cL\right] + 0.69C_L\left(rL + 0.65R_S + 0.36Z_0\right)$$



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RLC model – Rule of Thumb

- Transmission line effects should be considered when the rise or fall time of the input signal (t_r ,t_f) is smaller than the time-of-flight of the transmission line (t_{flight}).

$$t_r\ (t_f) << 2.5\ t_{flight}$$

- Transmission line effects should only be considered when the total resistance of the wire is limited:

$$R < 5\ Z_0$$

- The transmission line is considered lossless when the total resistance is substantially smaller than the characteristic impedance

$$R < Z_0/2$$

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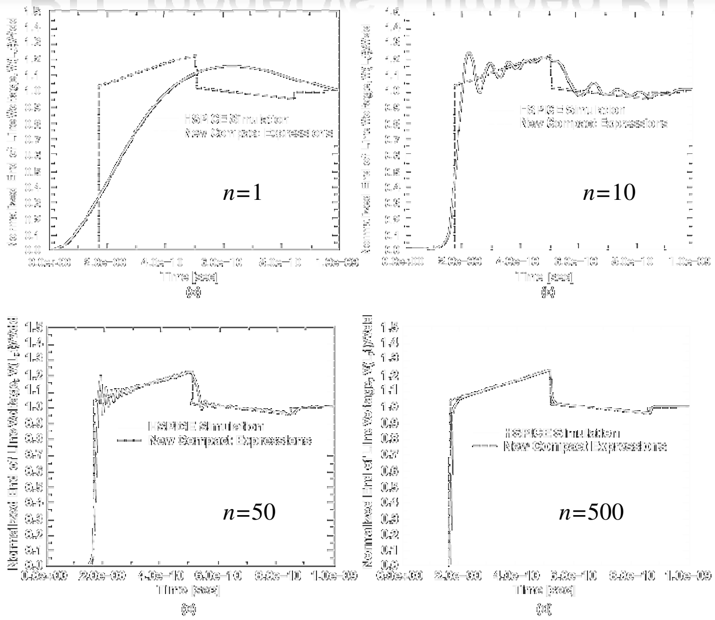
RLC model vs Lumped RLC



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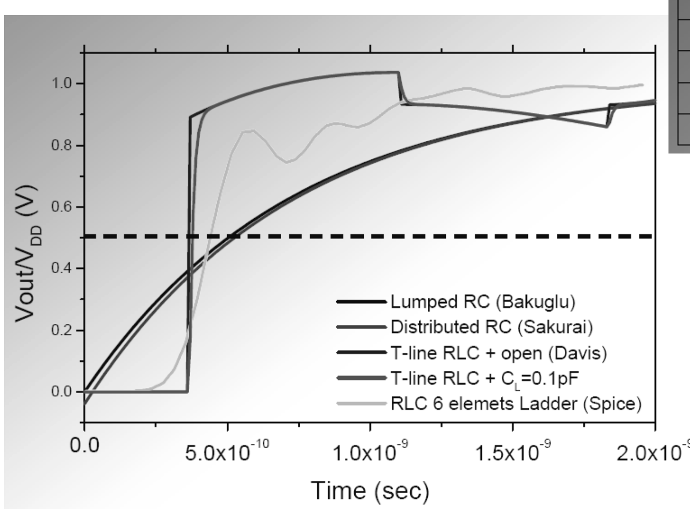
RLC step response



R_{tr}	130Ω
R	30Ω
C	1pF
L	15nH
Z_0	122Ω
l	3cm

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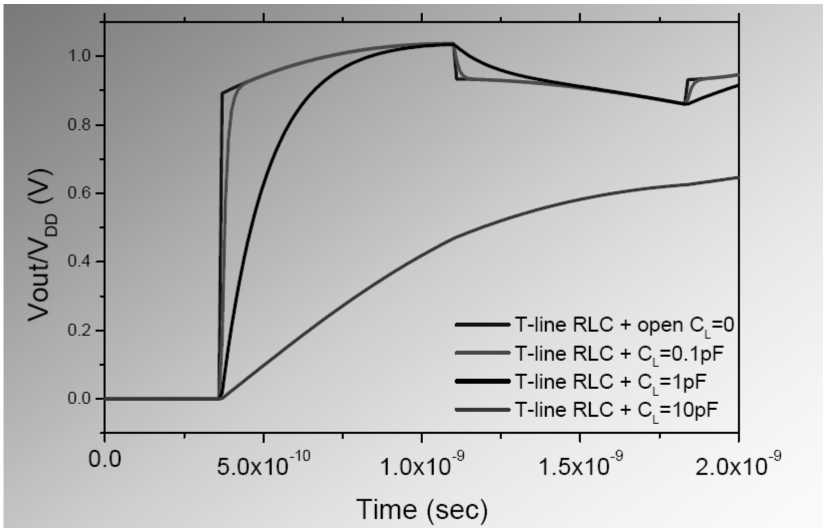
RLC Line – Load Capacitance



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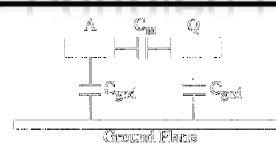
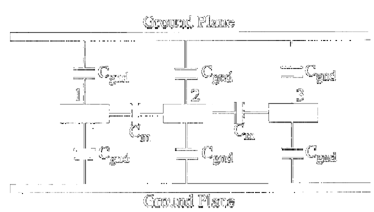
RLC model – Coupled Lines

2 coupled distributed RLC interconnects
A (active) and Q (quiet).



$$V_A(\ell, t) = V_{fin}(\ell, t, l = l_s - l_m, c = c_{gnd} + 2c_m)$$

3 coupled distributed RLC interconnects



$$V_A(\ell, t) = \frac{4}{3} V_{fin}\left(\ell, t, l = \frac{1}{(2c_{gnd} + 3c_m)v^2}, c = 2c_{gnd} + 3c_m\right) - \frac{1}{3} V_{fin}\left(\ell, t, l = \frac{1}{2c_{gnd}v^2}, c = 2c_{gnd}\right)$$

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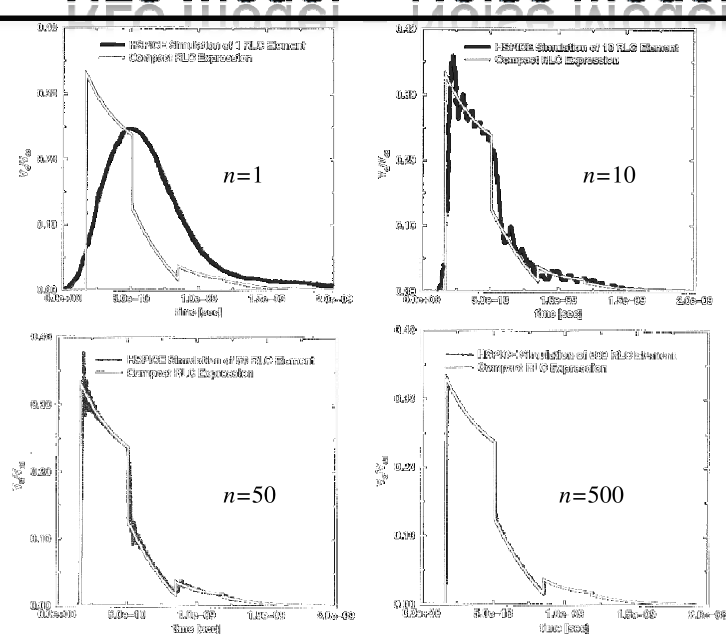
RLC model – Noise Model



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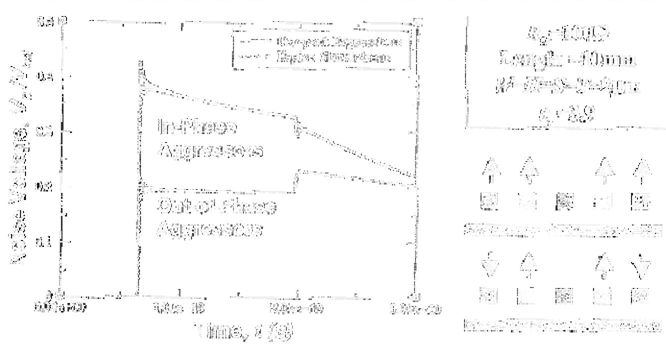
RLC model – Noise Model



The worst case crosstalk is when all aggressors switch in phase!!

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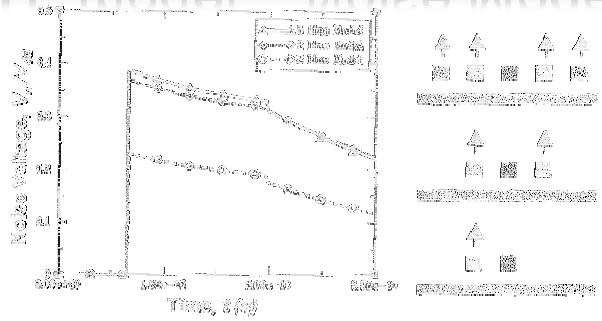
RLC model – Noise Model



WITH GROUND PLANE for aggressor have smaller IMPACT

2 line RC:

$$\frac{V_P}{V_{dd}} \approx \frac{1}{2}\frac{C_m}{C+C_m}$$

2 line RLC:

$$\frac{V_P}{V_{dd}} \approx \frac{\pi}{4}\frac{C_m}{C+C_m}$$

3 line RLC:

$$\frac{V_P}{V_{dd}} \approx \frac{\pi}{3}\frac{C_m}{\frac{4}{3}C+C_m}$$

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